Answer of First Semester Final Exam

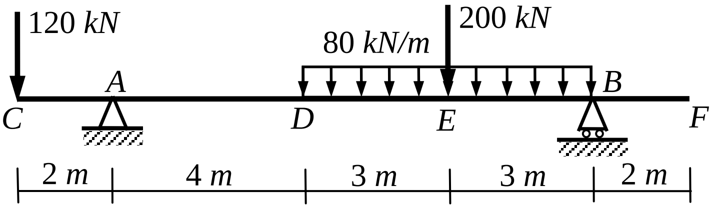
Question (1): (12 Marks)

For the shown beam, using the **double integration method**, determine:

(a) the deflections at *C*, *D* and *F*
(b) the slopes at *C* and *D*
and sketch the elastic curve of the beam.

$EI = 0.2\times10^9\ N.m^2$

$EI = 0.2\times10^6\ kN.m^2$



Solution:

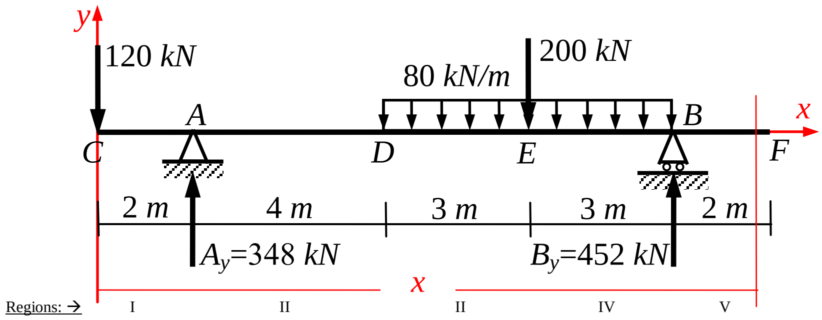
Reactions:

$-120\,(12) + A_y\,(10) - 680\,(3) = 0$

$\rightarrow A_y = 348\ kN$

$348 + B_y - 120 - 680 = 0$

$\rightarrow B_y = 452\ kN$



$$M = -120x\,\big|_{I} + 348(x-2)\,\big|_{II} - 80(x-6)^2/2\,\big|_{III} - 200(x-9)\,\big|_{IV} + 452(x-12) + 80(x-12)^2/2\,\big|_{V}$$

$$EI\,y'' = -120x\,\big|_{I} + 348(x-2)\,\big|_{II} - 40(x-6)^2\,\big|_{III} - 200(x-9)\,\big|_{IV} + 452(x-12) + 40(x-12)^2\,\big|_{V}$$

$$EI\,y' = -60x^2\,\big|_{I} + 174(x-2)^2\,\big|_{II} - 40(x-6)^3/3\,\big|_{III} - 100(x-9)^2\,\big|_{IV} + 226(x-12)^2 + 40(x-12)^3/3\,\big|_{V} + C_1$$

$$EI\,y = -20x^3\,\big|_{I} + 58(x-2)^3\,\big|_{II} - 10(x-6)^4/3\,\big|_{III} - 100(x-9)^3/3\,\big|_{IV} + 226(x-12)^3/3 + 10(x-12)^4/3\,\big|_{V} + C_1\,x + C_2$$

Boundary Conditions:

At $x = 2\ m$, $y = 0$ $\rightarrow 0 = -20(2)^3 + C_1\,(2) + C_2$ $\rightarrow$ $2C_1 + C_2 = 160$

At $x = 12\ m$, $y = 0$ $\rightarrow 0 = -20(12)^3 + 58(10)^3 - 10\,(6)^4/3 - 100(3)^3/3 + 0 + 0 + 12C_1 + C_2$

$\rightarrow$ $12C_1 + C_2 = -18220$ $C_1 = -1838$ and $C_2 = 3836$

So, the general equation of the deflection *y* at any distance *x* is,

$$EI\,y = -20x^3\,\big|_{I} + 58(x-2)^3\,\big|_{II} - 10(x-6)^4/3\,\big|_{III} - 100(x-9)^3/3\,\big|_{IV} + 226(x-12)^3/3 + 10(x-12)^4/3\,\big|_{V} - 1838\,x + 3836$$

(a) the deflection at *C* (*x*=0): in Region I:

$EI\,y_C = -20(0)^3 - 1838(0) + 3836 = +3836$

$y_C = 3836 / (0.2\times10^6) = 0.01918\ m = +19.18\ mm$ — **$y_C = 19.2\ mm\ \uparrow$**

the deflection at *D* (*x*=6): in Region II:

$EI\,y_D = -20(6)^3 + 58(4)^3 - 1838(6) + 3836 = -7800$

$y_D = -7800 / (0.2\times10^6) = -0.039\ m = -39.0\ mm$ — **$y_D = 39.0\ mm\ \downarrow$**

the deflection at *E* (*x*=9): in Region III:

$EI\,y_E = -20(9)^3 + 58(7)^3 - 10(3)^4/3 - 1838(9) + 3836 = -7662$ $\rightarrow$ $y_E = -7662 / (0.2\times10^6) = -0.03831\ m = -38.31\ mm$ — **$y_E = 38.3\ mm\ \downarrow$**

the deflection at *F* (*x*=14): in Region V:

$EI\,y_F = -20(14)^3 + 58(12)^3 - 10(8)^4/3 - 100(5)^3/3 + 226(2)^3/3 + 10(2)^4/3 - 1838(14) + 3836$

$= +6284$

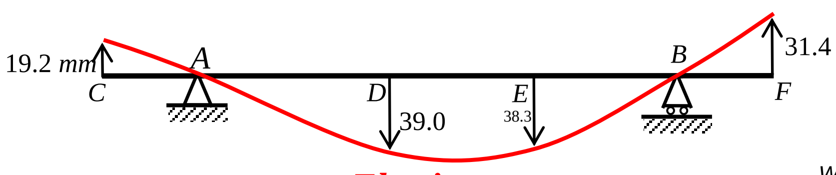
$y_F = 6284 / (0.2\times10^6) = -0.03142\ m = +31.42\ mm$ — **$y_F = 31.4\ mm\ \uparrow$**

(b) the slope at *C* (*x*=0): in Region I:

$EI\,y'_C = -60(0)^2 - 1838 = -1838$ $\rightarrow$ $\theta_C = y'_C = -1838 / (0.2\times10^6)$ — **$\theta_C = -0.00919\ rad = -0.53^\circ$** means ↻

the slope at *D* (*x*=6): in Region II:

$EI\,y'_D = -60(6)^2 + 174(4)^2 - 1838 = -1214$ $\rightarrow$ $\theta_D = -1214 / (0.2\times10^6)$ — **$\theta_D = -0.00607\ rad = -0.35^\circ$** means ↻



Elastic curve

With my best wishes
Dr. M. Abdel-Kader

Question (2): (12 Marks)

For the shown cantilever of rectangular cross-section 250 *mm* wide by ***h*** *mm* high, using the **moment-area method**, determine:

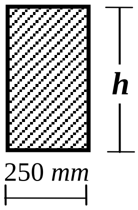
(a) the height ***h*** if the maximum deflection is not to exceed 10 *mm*

(b) the deflection at *C* (*use the calculated* ***h***)

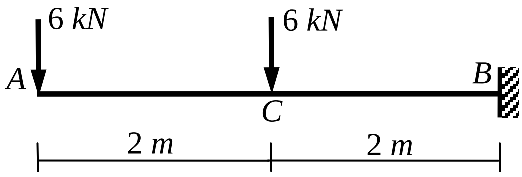
(c) the slope at *A* (*use the calculated* ***h***)

and sketch the elastic curve of the cantilever.

$E = 9\ GPa$



Cross-section



Solution:

$E = 9\ GPa = 9\times10^{9}\ N/m^2 = 9\times10^{6}\ kN/m^2$

The maximum deflection $\delta_{\max}$ will occur at the free end *A* which is equal to the deviation of the point *A* above the tangent to the elastic curve at *B* (which is the center line of the cantilever), then

(a) $\delta_{\max} = \delta_A = t_{A/B} = \dfrac{1}{EI}\left[Area_{AB}.\bar{X}_A\right]$

$$t_{A/B} = \frac{1}{EI}\left[\begin{array}{l}(-\tfrac{1}{2}\times2\times12)(4/3)+(-2\times12)(3)\\ +(-\tfrac{1}{2}\times2\times24)(10/3)\end{array}\right]$$

$$= \frac{-168}{EI} = -0.01$$

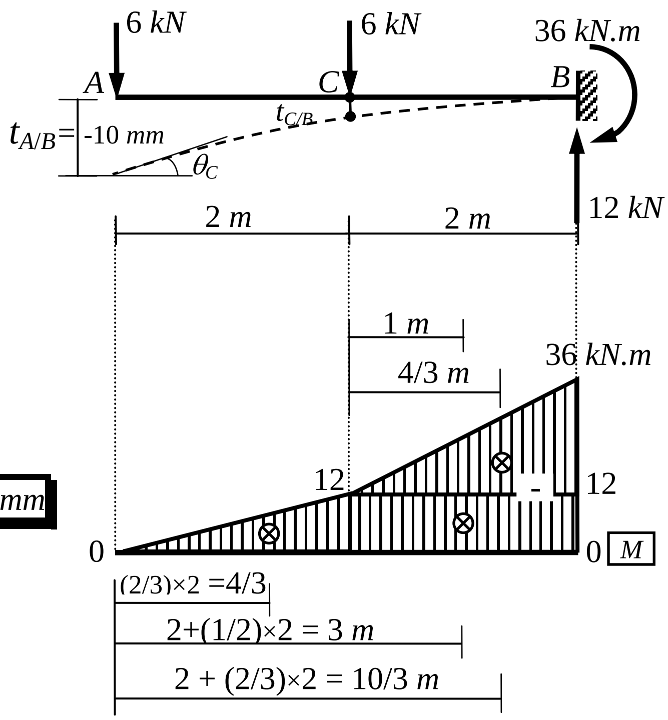
$$EI = \frac{-168}{-0.01} = 16800$$

$$9\times10^{6}\left(\frac{0.25h^3}{12}\right) = 16800$$

$$h^3 = \frac{16800\times12}{0.25\times9\times10^{6}} = 0.0896$$

$$\therefore\ h = 0.4475\ m = 447.5\ mm$$

$h = 447.5\ mm$



(b) $\delta_C = t_{C/B} = \dfrac{1}{EI}\left[Area_{CB}.\bar{X}_C\right]$

$$= \frac{1}{EI}\left[(-2\times12)(1)+(-\tfrac{1}{2}\times2\times24)(4/3)\right] = \frac{-56}{EI}$$

$$= \frac{-56}{9\times10^{6}\left(\frac{0.25\times0.4475^3}{12}\right)}\ 0.00333\ m = 3.33\ mm$$

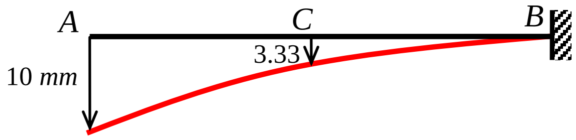
$\delta_C = 3.33\ mm \downarrow$

(c) $\theta_{AB} = \theta_A - \theta_B = \theta_A - 0 = \theta_A = \dfrac{1}{EI}\left[Area_{AB}\right]$

$$\theta_A = \frac{1}{EI}\left[(-\tfrac{1}{2}\times2\times12)+(-2\times12)+(-\tfrac{1}{2}\times2\times24)\right]$$

$$= \frac{-60}{EI} = \frac{-60}{9\times10^{6}\left(\frac{0.25\times0.4475^3}{12}\right)} = 0.00357\ rad = -0.2^{o}$$

$\theta_A = -\ 0.00357\ rad = -\ 0.2^{o}$ means ↻



Elastic curve

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Question (3): (12 Marks)

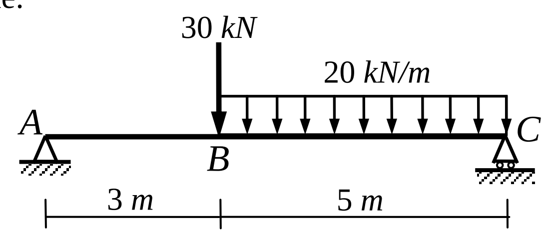
For the shown beam, using the **conjugate beam method**, determine:

(a) the slopes at ***A*** and ***B***.

(b) the deflection at ***B***.

and sketch the elastic curve of the beam.

$E = 200\ GPa$ $\qquad I = 290\times10^6\ mm^4$



Solution:

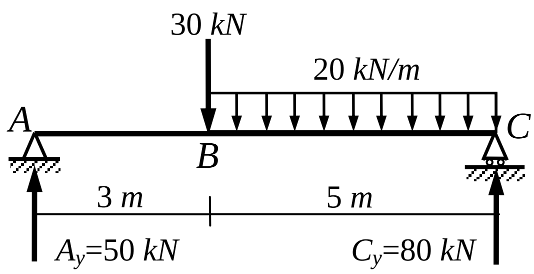
Reaction:

$+\circlearrowleft \sum M_C = 0:$

$A_y(8) - 30(5) - 20{\times}5(2.5) = 0 \rightarrow A_y = 50\ kN \uparrow$

$+\uparrow \sum F_y = 0:$

$A_y + C_y - 30 - 20{\times}5 = 0 \rightarrow C_y = 80\ kN \uparrow$

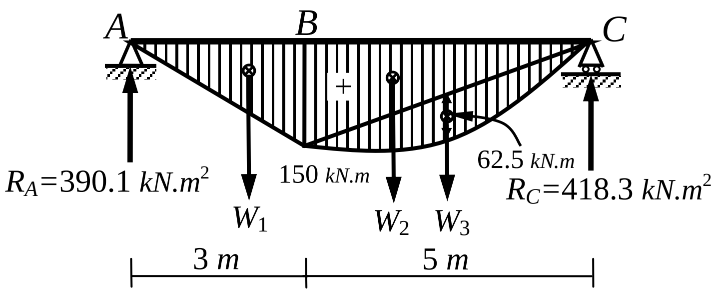


Construct the bending moment diagram of the real beam.
The resulting moment diagram is then loaded to the conjugate beam.
For the conjugate beam, determine the elastic reaction (R_A and R_C) at supports.

$W_1 = \frac{1}{2}\times3\times150 = 225\ kN.m^2$

$W_2 = \frac{1}{2}\times5\times150 = 375\ kN.m^2$

$W_3 = \frac{2}{3}\times5\times62.5 = 208.333\ kN.m^2$



$+\circlearrowleft \sum M_C = 0$

$R_A(8) - W_1(5+1) - W_2(2\times5/3) - W_3(5/2) = 0$

$8R_A = 225(6) + 375(10/3) + (208.333)(2.5) \rightarrow R_A = 390.1\ kN.m^2$

$+\uparrow \sum F_y = 0 \rightarrow R_C = 418.23\ kN.m^2$

$E = 200\ GPa = 200\times10^6\ kN/m^2 \qquad I = 290\times10^6\ mm^4 = 290\times10^{-6}\ m^4 \qquad EI = 58000\ kN.m^2$

(a) the slope at *A*

$\theta_A = R_A / EI = 390.1 / 58000 = 0.0067\ rad = 0.39^o$

$\boxed{\theta_A = 0.39^o}$ means ↻

the slope at *B*

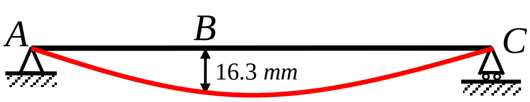
$\theta_B = Shear\ at\ B / EI = (R_A - W_1) / 58000 = (390.1 - 225) / 58000 = 0.00285\ rad = 0.163^o$

$\boxed{\theta_B = 0.163^o}$ means ↻

(b) the deflection at *B*

$\delta_B = Moment\ at\ B / EI = (R_A\times3 - W_1\times1) / 58000 = 945.3 / 58000 = 0.0163\ m = 16.3\ mm$

$\boxed{\delta_B = 16.3\ mm \downarrow}$



Elastic curve

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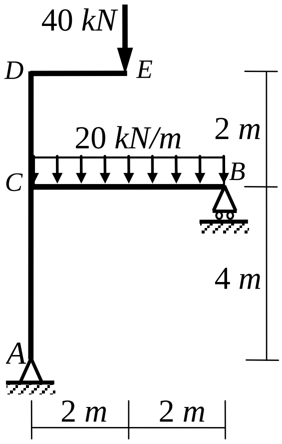
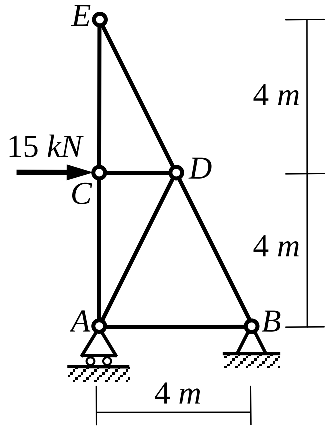
Question (4): (12 Marks)

For the shown frame and truss, using the **virtual work method**, determine the horizontal displacements at ***E*** (δ_{Eh}).

For the frame, $EI=50\times10^3\ kN.m^2$.
For the truss, assume that all members have the same axial rigidity $EA=10000\ kN$.



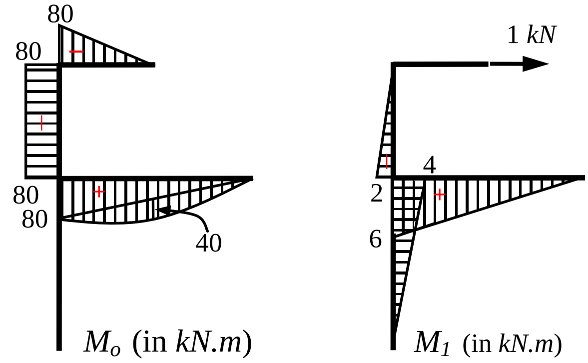


Solution:

(a) Horizontal displacement at *E*, δ_{Eh}

- Draw M_o-Diagram due to the applied loads.
- Apply a horizontal load of 1 *kN* at point *E* and draw M_1-Diagram due to this 1 *kN* load only.

then,



$$\delta_{Eh} = \int \frac{M_o M_1}{EI} dL$$

$$\delta_{Eh} = \frac{1}{EI}\left[\left(\tfrac{1}{2}\times4\times80\right)\left(\tfrac{2}{3}\times6\right)+\left(\tfrac{2}{3}\times4\times40\right)\left(\tfrac{1}{2}\times6\right)+\left(-2\times80\right)\left(-\tfrac{1}{2}\times2\right)\right]=\frac{1120}{EI}$$

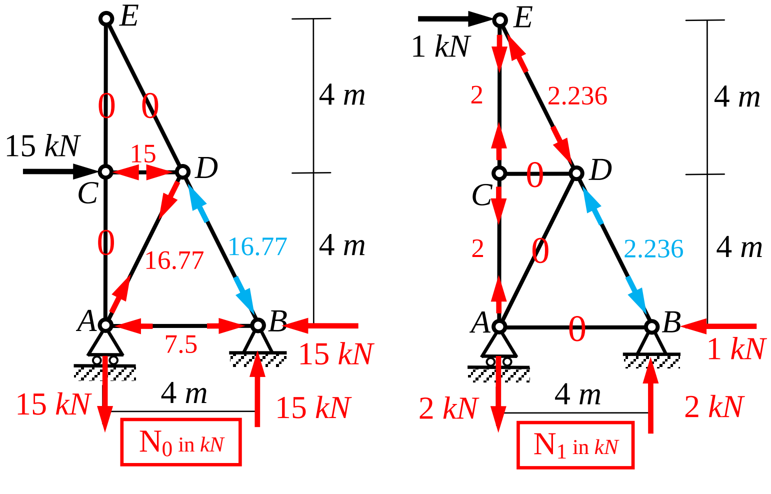
$$\delta_{Eh} = \frac{1120}{50\times10^3} = 0.0224\ m \quad = \quad 22.4\ mm$$

$\therefore$ $\delta_{Eh} = 22.4\ mm \rightarrow$

(b) Horizontal displacement at *E*, δ_{Eh}

- Calculate N_0 due to the applied loads.
- Apply a horizontal load of 1 *kN* at point *E* and calculate N_1 due to this 1 *kN* load only.

then,



$$\delta_{Eh} = \sum\frac{N_0 N_1 L}{EA} \quad = \quad \frac{(16.77\times2.236\times4.472)}{10000} = 0.0168\ m = 16.8\ mm$$

$\therefore$ $\delta_{Eh} = 16.8\ mm \rightarrow$

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Dr. M. Abdel-Kader

Question (5): (12 Marks)

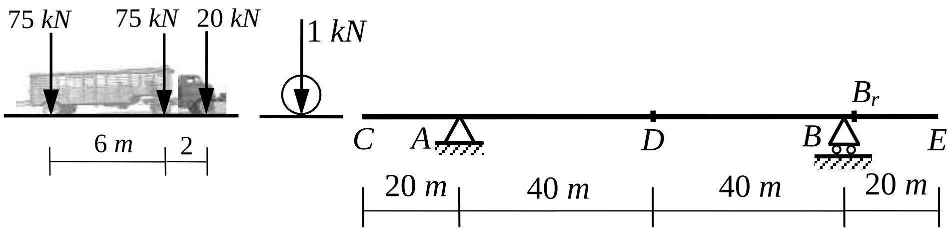
For the shown beam, draw the influence lines for:

(a) the reactions A_y, B_y.

(b) the shear forces at the sections D and B_r

(c) the bending moments at the sections A and D.

Also, determine the maximum positive and negative moments at D caused by the shown moving truck.

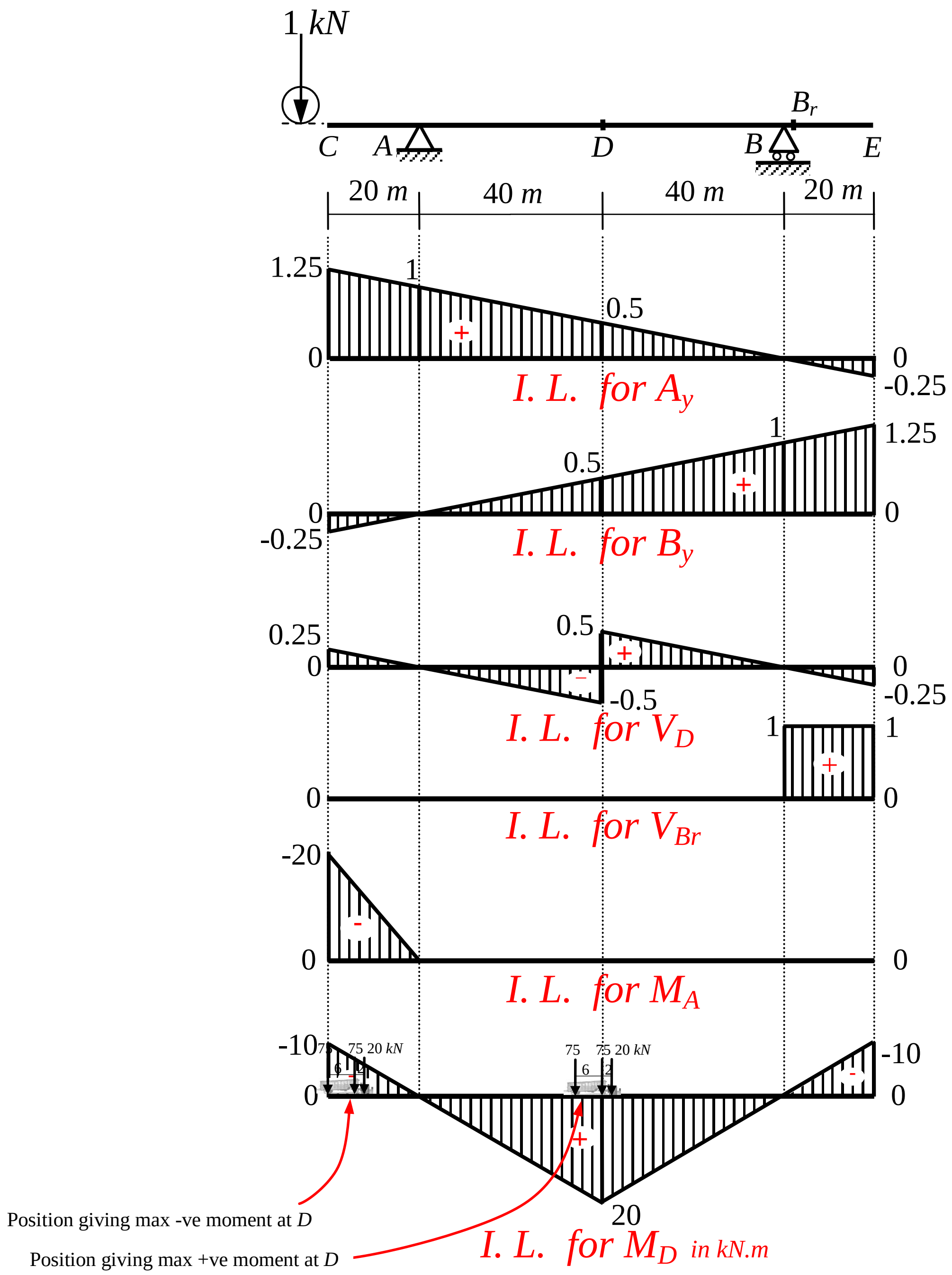


Solution:

(a)

(b)

(c)



$M_{D\,max\,+ve} = 75(0.85\times20) + 75(20) + 20(0.95\times20) = 3155\ kN.m$

$M_{D\,max\,+ve} = 3155\ kN.m$

$M_{D\,max\,-ve} = 75(-10) + 75(0.7\times-10) + 20(0.6\times-10) = 1395\ kN.m$

$M_{D\,max\,-ve} = 1395\ kN.m$

With my best wishes

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