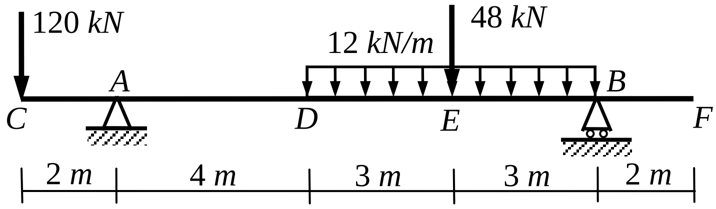
Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301**

Academic Year : **2016-2017**
Semester : **First**
Level : **3rd**
Time : **3 Hours**
Date : **17 / 1 / 2017**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Final Exam

Question (1): (12 Marks)

For the shown beam, using the **double integration method**, determine: the deflections at ***C***, ***D*** and ***F*** and the slope at ***C.*** Also, sketch the elastic curve of the beam. $EI = 2\times10^5\ kN.m^2$



Solution:

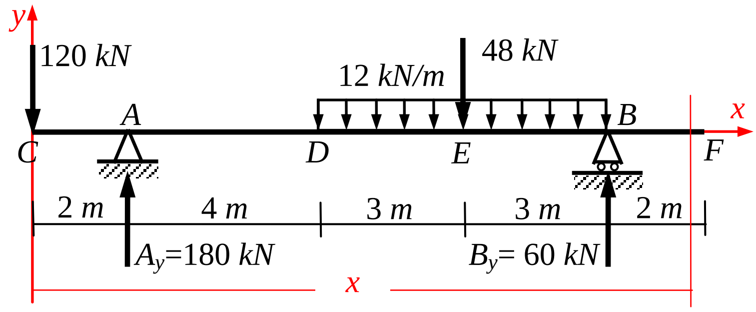
Reactions:

$-120\,(12) + A_y(10) - (72+48)(3) = 0$

$\rightarrow A_y = 180\ kN \uparrow$

$180 + B_y - 120 - 72 - 48 = 0$

$\rightarrow B_y = 60\ kN \uparrow$



Regions: → I II III IV V

$M = -120x \,|_{I} + 180(x-2) \,|_{II} - 12(x-6)^2/2 \,|_{III} - 48(x-9) \,|_{IV} + 60(x-12) + 12(x-12)^2/2 \,|_{V}$

$EI\,y'' = -120x \,|_{I} + 180(x-2) \,|_{II} - 6(x-6)^2 \,|_{III} - 48(x-9) \,|_{IV} + 60(x-12) + 6(x-12)^2 \,|_{V}$

$EI\,y' = -60x^2 \,|_{I} + 90(x-2)^2 \,|_{II} - 2(x-6)^3 \,|_{III} - 24(x-9)^2 \,|_{IV} + 30(x-12)^2 + 2(x-12)^3 \,|_{V} + C_1$

$EI\,y = -20x^3 \,|_{I} + 30(x-2)^3 \,|_{II} - 0.5(x-6)^4 \,|_{III} - 8(x-9)^3 \,|_{IV} + 10(x-12)^3 + 0.5(x-12)^4 \,|_{V} + C_1 x + C_2$

Boundary Conditions:

At $x = 2\ m$, $y = 0$ $\rightarrow 0 = -20(2)^3 + C_1(2) + C_2$ $\rightarrow$ $2C_1 + C_2 = 160$

At $x = 12\ m$, $y = 0$ $\rightarrow 0 = -20(12)^3 + 30(10)^3 - 0.5(6)^4 - 8(3)^3 + 0 + 0 + 12C_1 + C_2$

$\rightarrow$ $12C_1 + C_2 = 5424$ $C_1 = 526.4$ and $C_2 = -892.8$

So, the general equation of the deflection *y* at any distance *x* is,

$EI\,y = -20x^3 \,|_{I} + 30(x-2)^3 \,|_{II} - 0.5(x-6)^4 \,|_{III} - 8(x-9)^3 \,|_{IV} + 10(x-12)^3 + 0.5(x-12)^4 \,|_{V} + 526.4\,x - 892.8$

(a) the deflection at *C* (*x*=0): in Region I:

$EI\,y_C = -20(0)^3 - 30(0) + 526.4(0) - 892.8 = -892.8$

$y_C = -892.8 / (0.2\times10^6) = -0.00446\ m = -4.46\ mm$ $\quad$ $y_C = 4.46\ mm \downarrow$

the deflection at *D* (*x*=6): in Region II:

$EI\,y_D = -20(6)^3 + 30(4)^3 + 526.4(6) - 892.8 = -134.4$

$y_D = -134.4 / (0.2\times10^6) = -0.00067\ m = -0.67\ mm$ $\quad$ $y_D = 0.67\ mm \downarrow$

the deflection at *E* (*x*=9): in Region III:

$EI\,y_E = -20(9)^3 + 30(7)^3 - 0.5(3)^4 + 526.4\,(9) - 892.8 = -485.7$ $\rightarrow$ $y_E = -485.7 / (0.2\times10^6) = -0.00243\ m = -2.43\ mm$ $\quad$ $y_E = 2.43\ mm \downarrow$

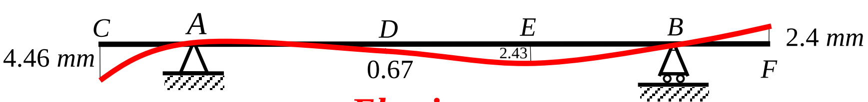
the deflection at *F* (*x*=14): in Region V:

$EI\,y_F = -20(14)^3 + 30(12)^3 - 0.5(8)^4 - 8(5)^3 + 10(2)^3 + 0.5(2)^4 + 526.4\,(14) - 892.8$
$= +476.8$

$y_F = 476.8 / (0.2\times10^6) = -0.002384\ m = +2.4\ mm$ $\quad$ $y_F = 2.4\ mm \uparrow$

(b) the slope at *C* (*x*=0): in Region I:

$EI\,y'_C = -60(0)^2 + 526.4 = +526.4$ $\rightarrow$ $\theta_C = y'_C = +526.4 / (0.2\times10^6)$ $\quad$ $\theta_C = +0.002632\ rad = 0.15^{o}$ ↻



Elastic curve

With my best wishes
Dr. M. Abdel-Kader

Question (2): (12 Marks)

For the shown beam, using the **moment-area method,** determine:

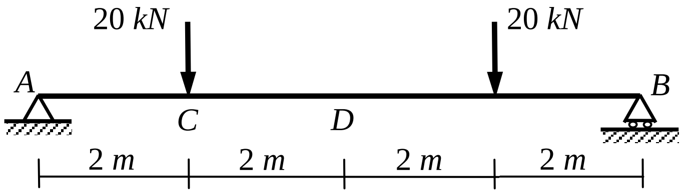
(a) the slope at ***A***

(b) the deflection at ***D***

(c) the deflection at ***C***

and sketch the elastic curve of the beam.

$E = 400\ GPa$ and $I = 1300\ cm^4$



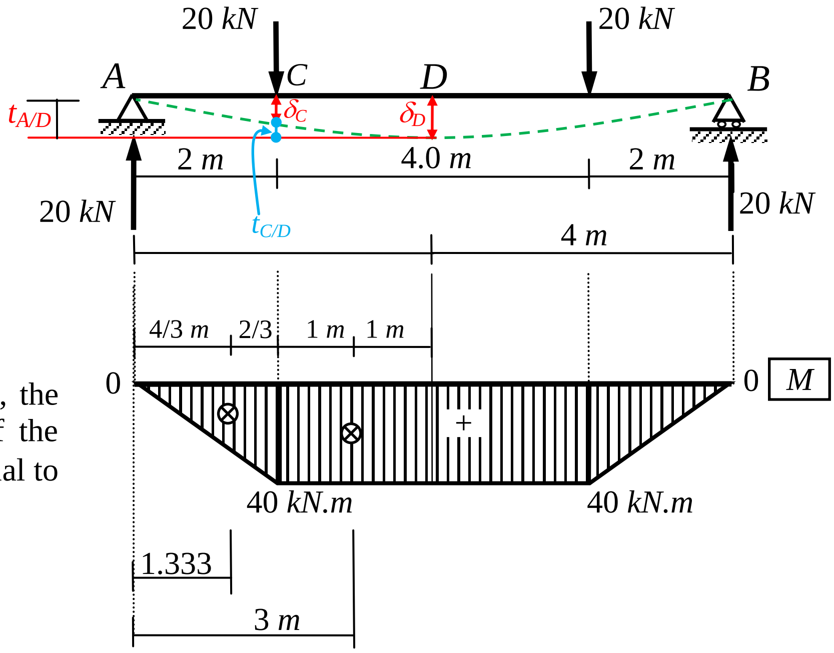
Solution:

$E = 400\ GPa = 400 \times 10^6\ kN/m^2$

$I = 1300\ cm^4 = 1300 \times 10^{-8}\ m^4$

$EI = (400 \times 10^6)(1300 \times 10^{-8})$

$= 5200\ kN.m^2$



(a) Slope at *A*

- Since the slope at ***D*** (θ_D) is equal to zero, the change in slope between the tangents of the elastic curve at points *A* and *B* (θ_{DA}) is equal to the slope at ***A*** (θ_A),

$$\theta_{DA} = \theta_D - \theta_A = 0 - \theta_A = -\theta_A$$

then

$$-\theta_A = \theta_{DA} = \frac{1}{EI}\left[Area_{AD}\right]$$

$$= \frac{1}{5200}\left[(\tfrac{1}{2} \times 2 \times 40) + (2 \times 40)\right] = \frac{120}{5200} = 0.023\ rads$$

$\theta_A = -0.023 \times 180/\pi = -1.32\ degrees$ $\qquad \therefore$ $\boxed{\theta_A = -1.32^o}$ ↻

(b) Deflection at *D*

- The deflection at ***D*** (δ_D) is equal to the deviation of the point ***A*** above the tangent to the elastic curve at ***D***, then

$$\delta_D = t_{A/D} = \frac{1}{EI}\left[Area_{AD}.\bar{X}_A\right] = \frac{1}{5200}\left[(\tfrac{1}{2} \times 2 \times 40)(4/3) + (2 \times 40)(3)\right] = 0.0564\ m$$

$= 0.0564 \times 1000 = 56.4\ mm$ $\qquad \therefore$ $\boxed{\delta_D = 56.4\ mm}$ ↓

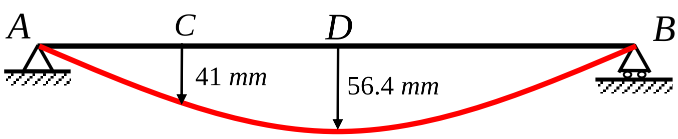
(b) Deflection at *C*

- The deflection at ***C*** (δ_C) is equal to the deflection at ***D*** (δ_D) minus the deviation of the point ***C*** above the tangent to the elastic curve at ***D*** ($t_{C/D}$), then

$$\delta_C = \delta_D - t_{C/D} = \delta_D - \frac{1}{EI}\left[Area_{CD}.\bar{X}_C\right]$$

$$= 0.0564 - \frac{1}{5200}\left[(2 \times 40)(1)\right] = 0.0564 - 0.01538 = 0.04102\ m$$

$= 0.04102 \times 1000 = 41.02\ mm$ $\qquad \therefore$ $\boxed{\delta_C = 41\ mm}$ ↓



Elastic curve

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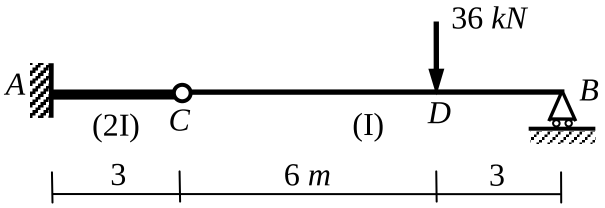
Question (3): (12 Marks)

For the shown beam of variable cross-section (the relative moments of inertia are given between brackets), using the **conjugate beam method**, determine the slope at ***B*** and the deflections at ***C*** and ***D.*** Also, sketch the elastic curve of the beam. $EI = 20\times10^3\ kN.m^2$



Solution:

Reaction:

$+\circlearrowleft \sum M_C = 0:\ 36(6) - B_y(9) = 0 \quad \rightarrow \quad B_y = 24\ kN \uparrow$

$+\uparrow \sum F_y = 0: \quad B_y + C_y - 36 = 0 \quad \rightarrow \quad C_y = 12\ kN \uparrow$

Construct the bending moment diagram of the real beam. The resulting moment diagram is then loaded to the conjugate beam.

$W_1 = \frac{1}{2}\times3\times18 = 27\ kN.m^2$

$W_2 = \frac{1}{2}\times6\times72 = 216\ kN.m^2$

$W_3 = \frac{1}{2}\times3\times72 = 108\ kN.m^2$

For the conjugate beam, determine the elastic reactions (R_B and R_C) at the supports.

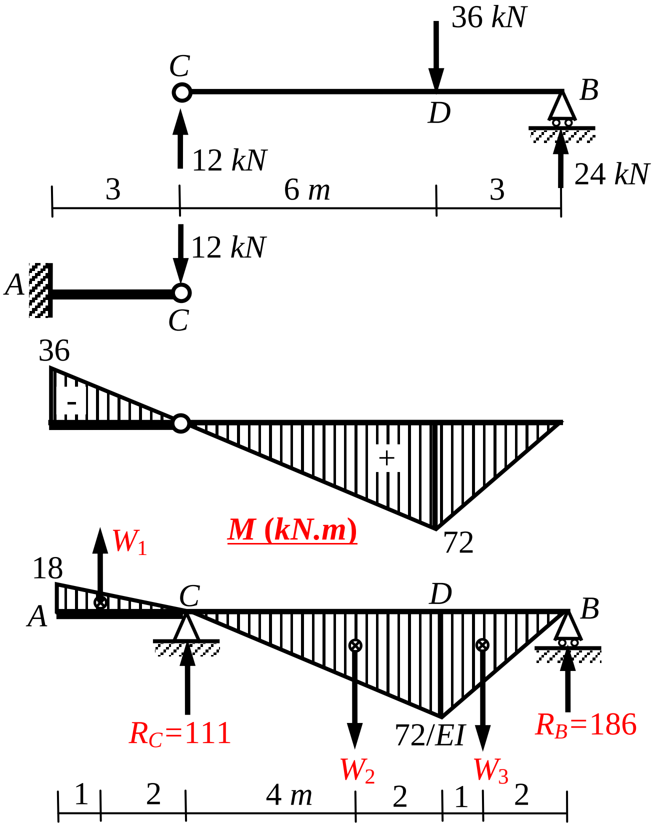
$+\circlearrowleft \sum M_C = 0$

$W_1(2) + W_2(4) + W_3(7) - R_B(9) = 0$

$9R_B = 27(2) + 216(4) + 108(7)$

$\rightarrow R_B = 186\ kN.m^2$

$+\uparrow \sum F_y = 0 \quad \rightarrow R_C = 111\ kN.m^2$



M/EI loaded on conjugate beam

Slope at *B*

$\theta_B = -R_B / EI = 186 / 20\times10^3 = -0.0093\ rad = -0.53^\circ$

$\theta_B = 0.53^\circ$ ↻

Deflection at *C*

$\delta_C = Moment\ at\ C / EI = (W_1\times2) / EI = 54 / 20\times10^3 = 0.0027\ m = 2.7\ mm$

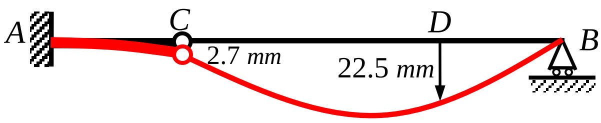
$\delta_C = 2.7\ mm \downarrow$

Deflection at *D*

$\delta_D = Moment\ at\ D / EI = (R_B\times3 - W_3\times1) / EI = (186\times3 - 108\times1) / 20\times10^3$

$= 0.0225\ m = 22.5\ mm$

$\delta_D = 22.5\ mm \downarrow$



Elastic curve

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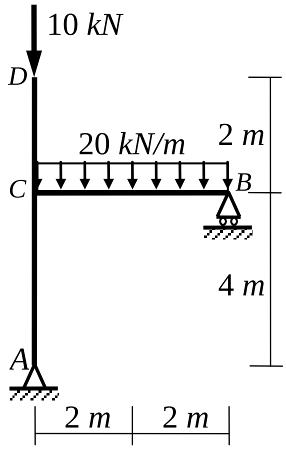
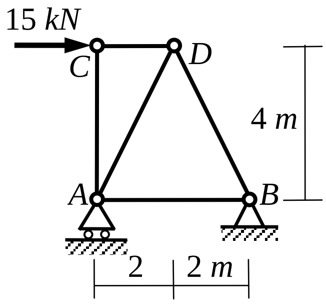
Question (4): (12 Marks)

For the shown frame and truss, using the **virtual work method**, determine the horizontal displacements at ***D*** (δ_{Dh}).

For the frame, $EI = 50\times10^3\ kN.m^2$.

For the truss, assume that all members have the same axial rigidity $EA = 10000\ kN$.

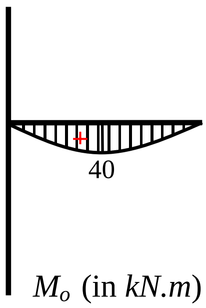




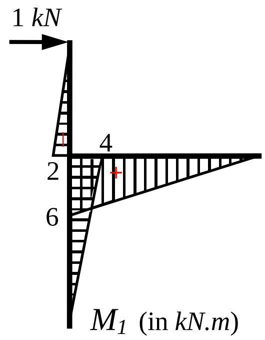
Solution:

(a) Horizontal displacement at *D*, δ_{Dh}

- Draw M_o-Diagram due to the applied loads.
- Apply a horizontal load of 1 kN at point D and draw M_1-Diagram due to this 1 kN load only. then,



M_o (in $kN.m$)



M_1 (in $kN.m$)

$$\delta_{Dh} = \int \frac{M_o M_1}{EI} dL$$

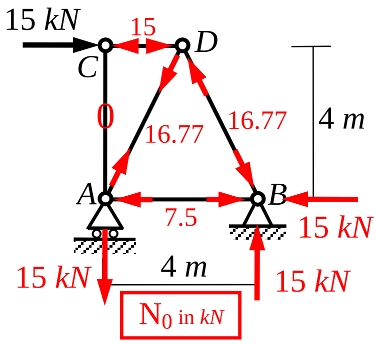
$$\delta_{Dh} = \frac{1}{EI}\left[\left(\tfrac{2}{3}\times 4\times 40\right)\left(\tfrac{1}{2}\times 6\right)\right] = \frac{320}{EI}$$

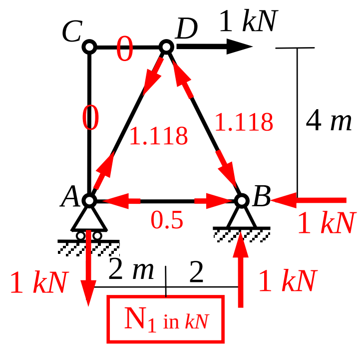
$$\delta_{Dh} = \frac{320}{50\times10^3} = 0.0064\ m \quad = \quad 6.4\ mm$$

$\therefore$ $\boxed{\delta_{Dh} = \ 6.4\ mm \rightarrow}$

(b) Horizontal displacement at *D*, δ_{Dh}

- Calculate N_0 due to the applied loads.
- Apply a horizontal load of 1 kN at point D and calculate N_1 due to this 1 kN load only.





For N_0

Joint *D*: $+\rightarrow \sum F_x = F_{DB}(0.4472) - F_{DA}(0.4472) + 15 = 0$

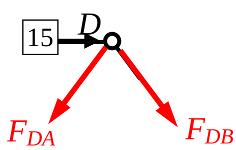
$\therefore\ F_{DB} - F_{DA} = -33.54$ .... (1)

$+\uparrow \sum F_y = -F_{DB}(0.8944) - F_{DA}(0.8944) = 0$

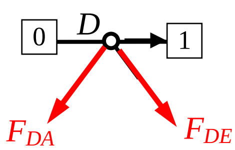
$\therefore\ F_{DB} = -F_{CA}$ ...... (2)

From (1) in (2) $\quad F_{DA} = 16.77$ and $F_{DB} = -16.77$

$\therefore$ $\boxed{F_{DA} = 16.77\ T}$ and $\boxed{F_{DB} = 16.77\ C}$ ...... and so on



Joint D for N_0



Joint D for N_1

then,

$$\delta_{Dh} = \sum \frac{N_0 N_1 L}{EA}$$

$$= \frac{(2\times16.77\times1.118\times4.472 + 7.5\times0.5\times4)}{10000} = 0.01827\ m = 18.27\ mm$$

$\therefore$ $\boxed{\delta_{Dh} = \ 18.3\ mm \rightarrow}$

With my best wishes

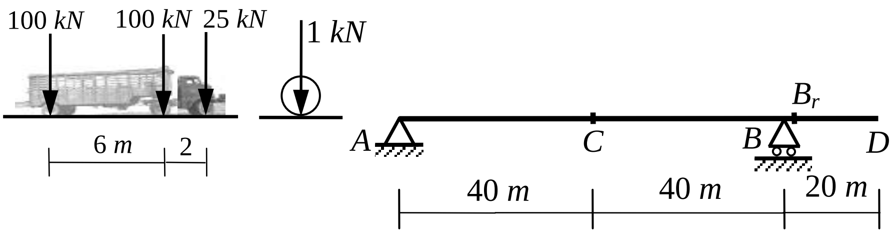
Dr. M. Abdel-Kader

Question (5): (12 Marks)

For the shown beam, draw the influence lines for:

(a) the reactions A_y, B_y.
(b) the shear forces at C and B_r
(c) the bending moment at C

Also, determine the maximum positive moment at C caused by the shown moving truck.

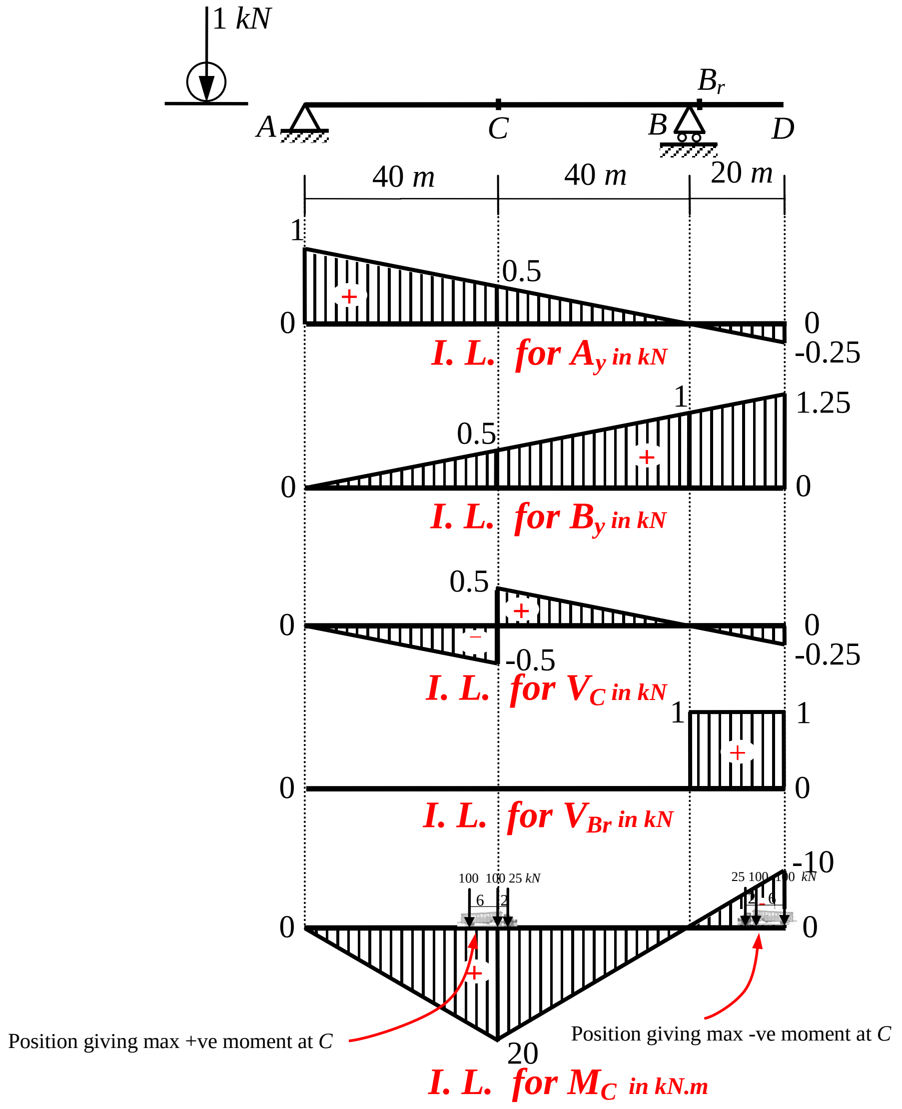


Solution:

(a)

(b)

(c)



$M_{D\ max\ +ve} = 100(0.85\times20) + 100(20) + 25(0.95\times20) = 4175\ kN.m$

$M_{D\ max\ +ve} = 4175\ kN.m$

$M_{D\ max\ -ve} = 100(-10) + 100(0.7\times-10) + 25(0.6\times-10) = 1850\ kN.m$

$M_{D\ max\ -ve} = 1855\ kN.m$

With my best wishes

Dr. M. Abdel-Kader