

### Answer of Final Exam

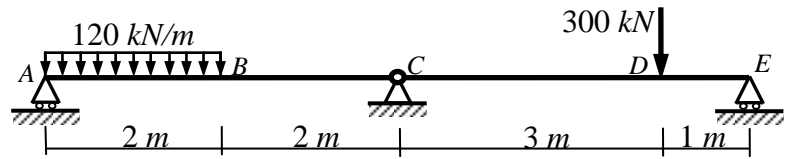
Total Marks: 60

No. of Questions: 5 (Attempt all questions)

#### Question (1): (12 Marks)

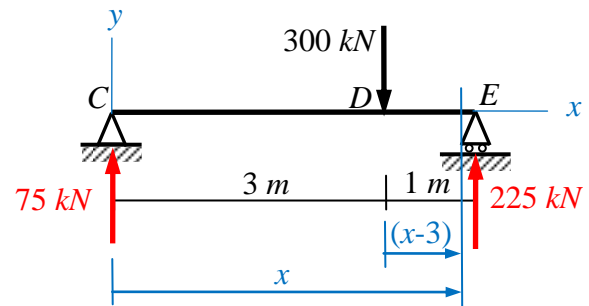
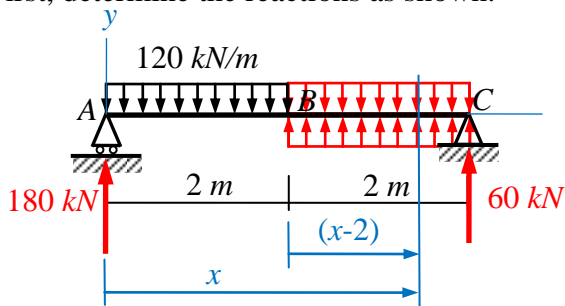
For the shown beam, using the double integration method:

- Determine the deflections at B and the mid-span CE.
  - Determine the slopes just to the left and the right of C.
  - Sketch the elastic curve of the beam.
- $EI = 2 \times 10^7 \text{ N.m}^2$



#### Solution:

- Note that there is an intermediate hinge above the intermediate support at C. This intermediate hinge separates the beam to two parts (AC and CE) as shown.
- First, determine the reactions as shown.



$$EIy'' = M$$

$$EIy'' = 180x - 120x^2/2 + 120(x-2)^2/2$$

$$EIy' = 90x^2 - 20x^3 + 20(x-2)^3 + C_1$$

$$EIy = 30x^3 - 5x^4 + 5(x-2)^4 + C_1x + C_2$$

#### Boundary Conditions:

At  $x = 0, y = 0 \rightarrow C_2 = 0$

At  $x = 4, y = 0 \rightarrow$

$$0 = 30(4)^3 - 5(4)^4 + 5(2)^4 + C_1(4) + 0 \rightarrow C_1 = -180$$

So, the general equation of the deflection  $y$  and slope  $y'(\theta)$  at any distance  $x$  is,

$$EIy' = 90x^2 - 20x^3 + 20(x-2)^3 - 180$$

$$EIy = 30x^3 - 5x^4 + 5(x-2)^4 - 180x$$

The deflection at B is (at  $x = 2 \text{ m}$ )

$$EI\delta_B = 30(2)^3 - 5(2)^4 - 180(2) = -200$$

$$\delta_B = -200/20000 = -0.01 \text{ m}$$

$$\delta_B = 10 \text{ mm} \downarrow$$

The slope at section just to the left of C is (at  $x = 4 \text{ m}$ )

$$EIy' = 90(4)^2 - 20(4)^3 + 20(2)^3 - 180 = 140$$

$$\theta_{c \text{ left}} = 140/20000 = 0.007 \text{ rad}$$

$$\theta_{c \text{ left}} = 0.401^\circ \curvearrowright$$

$$EIy'' = M$$

$$EIy'' = 75x - 300(x-3)$$

$$EIy' = 37.5x^2 - 150(x-3)^2 + C_3$$

$$EIy = 12.5x^3 - 50(x-3)^3 + C_3x + C_4$$

#### Boundary Conditions:

At  $x = 0, y = 0 \rightarrow C_4 = 0$

At  $x = 4, y = 0 \rightarrow$

$$0 = 12.5(4)^3 - 50(1)^4 + C_3(4) + 0 \rightarrow C_3 = -187.5$$

So, the general equation of the deflection  $y$  and slope  $y'(\theta)$  at any distance  $x$  is,

$$EIy' = 37.5x^2 - 150(x-3)^2 - 187.5$$

$$EIy = 12.5x^3 - 50(x-3)^3 - 187.5x$$

The deflection at mid-span is (at  $x = 2 \text{ m}$ )

$$EI\delta_{\text{mid-span}} = 12.5(2)^3 - 187.5(2) = -275$$

$$\delta_{\text{mid-span}} = -275/20000 = -0.01375 \text{ m}$$

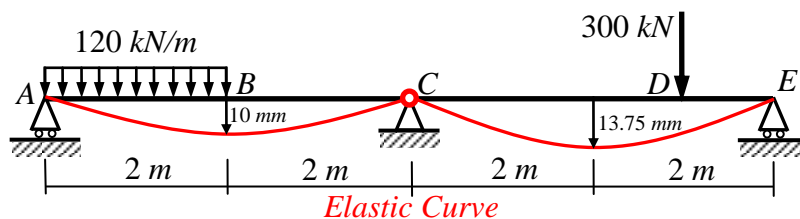
$$\delta_{\text{mid-span}} = 13.75 \text{ mm} \downarrow$$

The slope at section just to the right of C is (at  $x = 0$ )

$$EIy' = 37.5(0)^2 - 187.5 = -187.5$$

$$\theta_{c \text{ right}} = -187.5/20000 = -0.0094 \text{ rad}$$

$$\theta_{c \text{ right}} = 0.537^\circ \curvearrowright$$

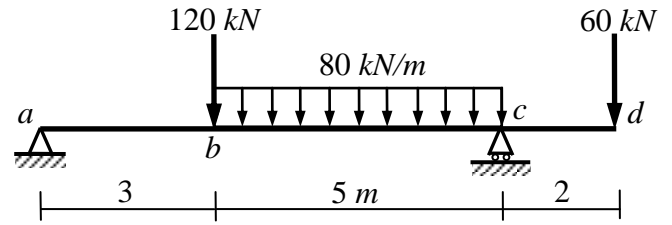


**Question (2): (12 Marks)**

For the shown beam, using the **moment-area method**:

- (a) Determine the slope at *a*.
- (b) Determine the deflections at *b* and *d*.
- (c) Sketch the elastic curve of the beam.

$EI = 150 \text{ MN.m}^2$



**Solution:**

The bending moment diagram may be drawn as shown.

**(a) The slope at *a***

$$\theta_a = \frac{t_{c/a}}{8}$$

Apply the second moment-area theorem, then

$$t_{c/a} = \frac{1}{EI} [Area_{ac} \cdot \bar{X}_c]$$

$$= \frac{1}{EI} \left[ \left(\frac{1}{2} \times 3 \times 600\right)(6) + \left(\frac{1}{2} \times 5 \times 600\right)\left(\frac{10}{3}\right) + \left(\frac{2}{3} \times 5 \times 250\right)(2.5) + \left(-\frac{1}{2} \times 8 \times 120\right)\left(\frac{8}{3}\right) \right]$$

$$= \frac{33610}{3EI} = \frac{33610}{3 \times 150000} = 3351/45000 = 0.07469 \text{ m}$$

$$\therefore \theta_a = \frac{t_{c/a}}{8} = \frac{0.07469}{8} = 0.009336 \text{ rad} = 0.535^\circ$$

$$\theta_a = 0.535^\circ \curvearrowright$$

**(b) The deflection at *b***

The deflection at  $b = \delta_b = bb'' - b'b'' = (3/8) t_{c/a} - t_{b/a}$

Applying the second moment-area theorem, then

$$t_{b/a} = \frac{1}{EI} [\text{First moment of area of M - diagram between } a \text{ and } b \text{ about } b]$$

$$= \frac{1}{EI} [Area_{ab} \cdot \bar{X}_b] = \frac{1}{EI} \left[ \left(\frac{1}{2} \times 3 \times 600\right)(1) + \left(-\frac{1}{2} \times 3 \times 45\right)(1) \right] = \frac{3330}{4EI} = \frac{3330}{4 \times 150000} = 0.00555 \text{ m}$$

$$\therefore \delta_b = bb'' - b'b'' = (3/8) t_{c/a} - t_{b/a} = (3/8)(0.07469) - 0.00555 = 0.02246 \text{ m}$$

$$\delta_b = 22.46 \text{ mm} \downarrow$$

**(c) The deflection at *d***

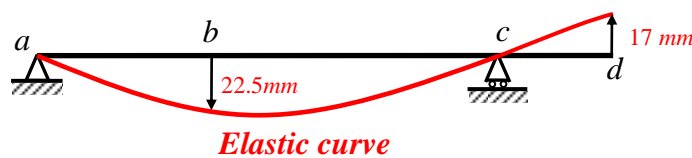
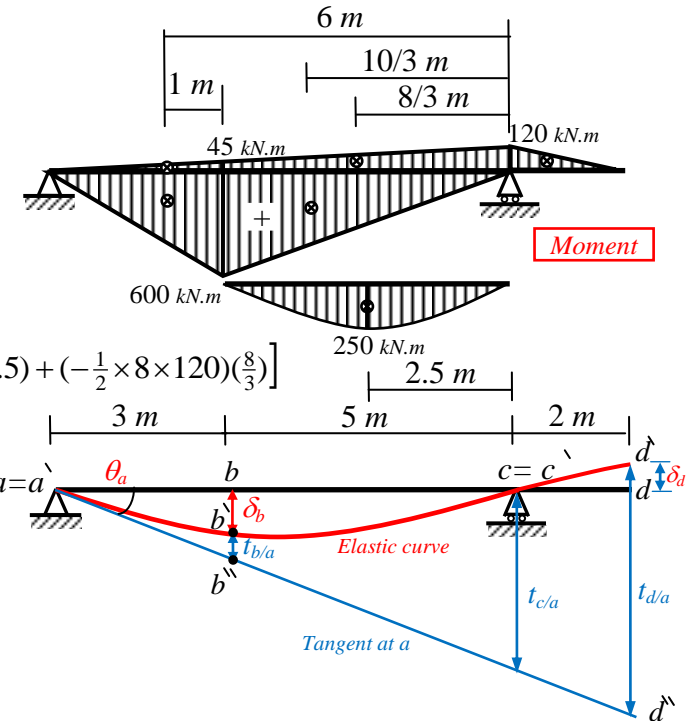
$$t_{d/a} = \frac{1}{EI} [Area_{ad} \cdot \bar{X}_d]$$

$$= \frac{1}{EI} \left[ \left(\frac{1}{2} \times 3 \times 600\right)(8) + \left(\frac{1}{2} \times 5 \times 600\right)\left(\frac{16}{3}\right) + \left(\frac{2}{3} \times 5 \times 250\right)(4.5) + \left(-\frac{1}{2} \times 8 \times 120\right)\left(\frac{14}{3}\right) + \left(-\frac{1}{2} \times 2 \times 120\right)\left(\frac{4}{3}\right) \right]$$

$$= \frac{16550}{EI} = \frac{16550}{150000} = 331/3000 = 0.110333 \text{ m}$$

$$\therefore \delta_d = d'd'' - dd'' = t_{d/a} - (10/8) t_{c/a} = 0.110333 - (10/8)(0.07469) = 0.01697 \text{ m}$$

$$\delta_d = 16.97 \text{ mm} \uparrow$$



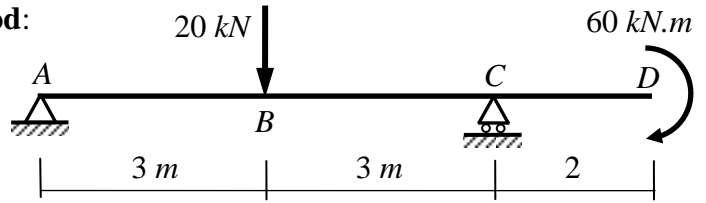
With my best wishes  
Dr. M. Abdel-Kader

**Question (3): (12 Marks)**

For the shown beam, using the **conjugate beam method**:

- (a) Determine the slope at **C**.
- (b) Determine the deflections at **B** and **D**.
- (c) Sketch the elastic curve of the beam.

$EI = 40 \times 10^3 \text{ kN.m}^2$

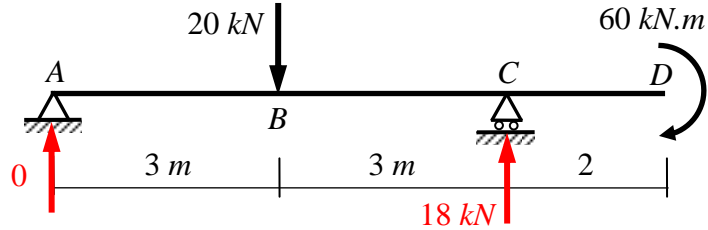


**Solution:**

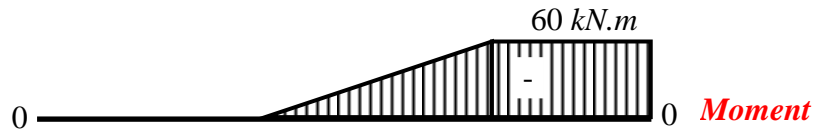
Reaction:

$+\circlearrowleft \sum M_A = 0$

$A_y(6) - 20 \times 3 + 60 = 0 \rightarrow A_y = 0$

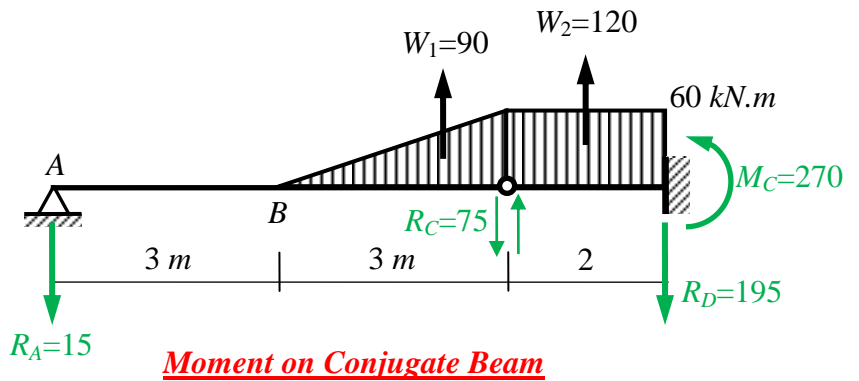


First construct the bending moment diagram of the real beam.



The resulting moment diagram is then loaded to the conjugate beam.

For the conjugate beam, determine the elastic reaction ( $R_C$ ,  $M_B$  and  $M_D$ ).



$W_1 = \frac{1}{2} \times 3 \times 60 = 90 \text{ kN.m}^2$

$W_2 = 2 \times 60 = 120 \text{ kN.m}^2$

$M_C$  for left part = 0

$R_A(6) - W_1(1) = 0 \rightarrow R_A = 15 \text{ kN.m}^2 \rightarrow R_C = 90 - 15 = 75 \text{ kN.m}^2$

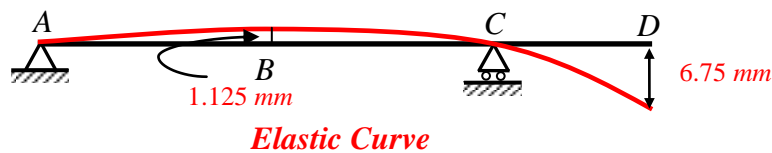
$M_B = -R_A(3) = -15(3) = -45 \text{ kN.m}^3$

$M_D = R_C(2) + W_2(1) = 75(2) + 120(1) = 270 \text{ kN.m}^3$

(a) Slope at C =  $R_C / EI = 75 / 40000 = 0.001875 \text{ rad} = 0.1074^\circ \therefore \theta_c = 0.11^\circ$

(b) Deflection at B =  $M_B / EI = -45 / 40000 = -0.001125 \text{ m} = -1.125 \text{ mm} \therefore \delta_B = 1.125 \text{ mm} \uparrow$

Deflection at D =  $M_D / EI = 270 / 40000 = 0.00675 \text{ m} = 6.75 \text{ mm} \therefore \delta_D = 6.75 \text{ mm} \downarrow$

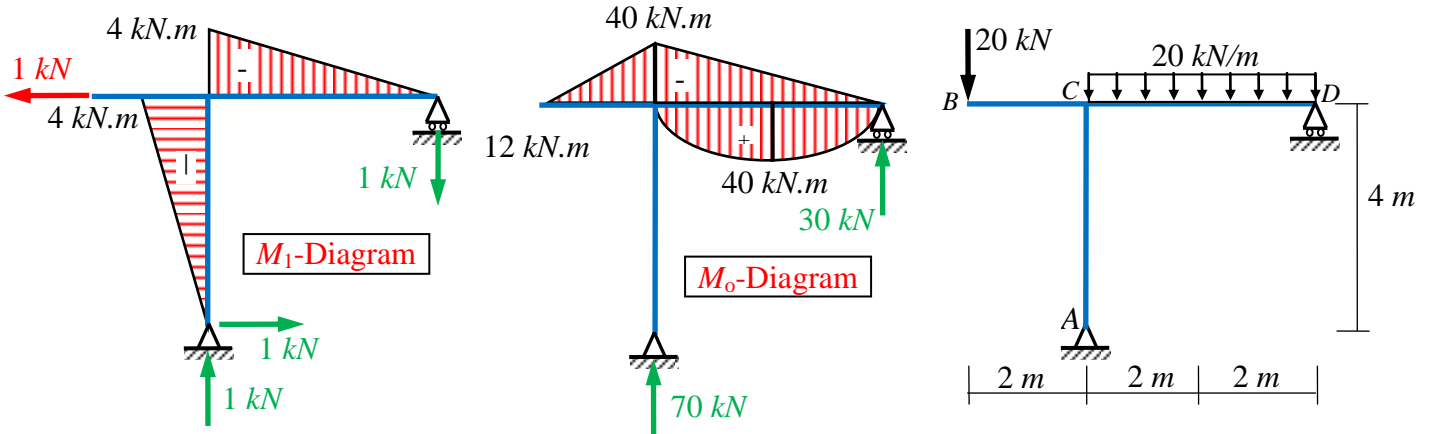


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**Question (4): (12 Marks)**

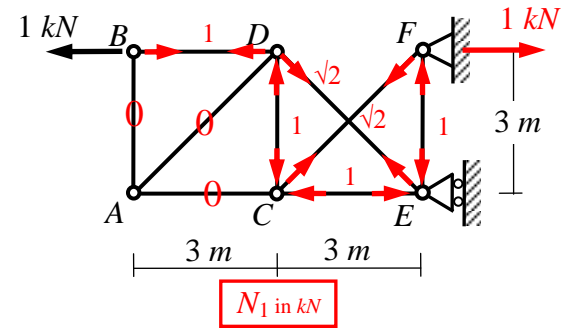
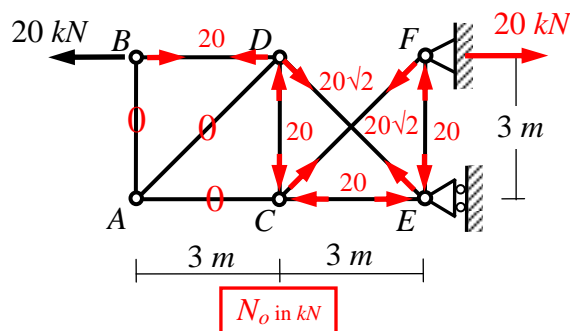
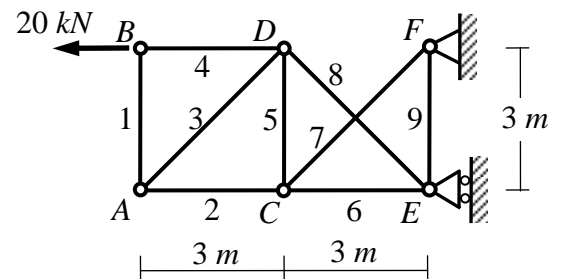
For the shown frame and truss, using the **virtual work method**, determine the horizontal displacements at B ( $\delta_{Bh}$ ). For the frame,  $EI = 20 \times 10^3 \text{ kN.m}^2$ . For the truss, assume that all members have the same axial rigidity  $EA = 30000 \text{ kN}$ .

**Solution:**



$$\delta_{Bh} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI} \left[ \left( -\frac{1}{2} \times 4 \times 40 \right) \left( -\frac{2}{3} \times 4 \right) + \left( \frac{2}{3} \times 4 \times 40 \right) \left( -\frac{1}{2} \times 4 \right) \right] = 0$$

$$\therefore \delta_{Bh} = 0$$



$$\delta_{Bh} = \sum \frac{N_o N_1 L}{EA}$$

$$= \frac{4(20 \times 1 \times 3) + 2(20\sqrt{2} \times \sqrt{2} \times 3\sqrt{2})}{30000} = 0.01931 \text{ m} = 19.31 \text{ mm}$$

$$\therefore \delta_{Bh} = 19.31 \text{ mm} \leftarrow$$

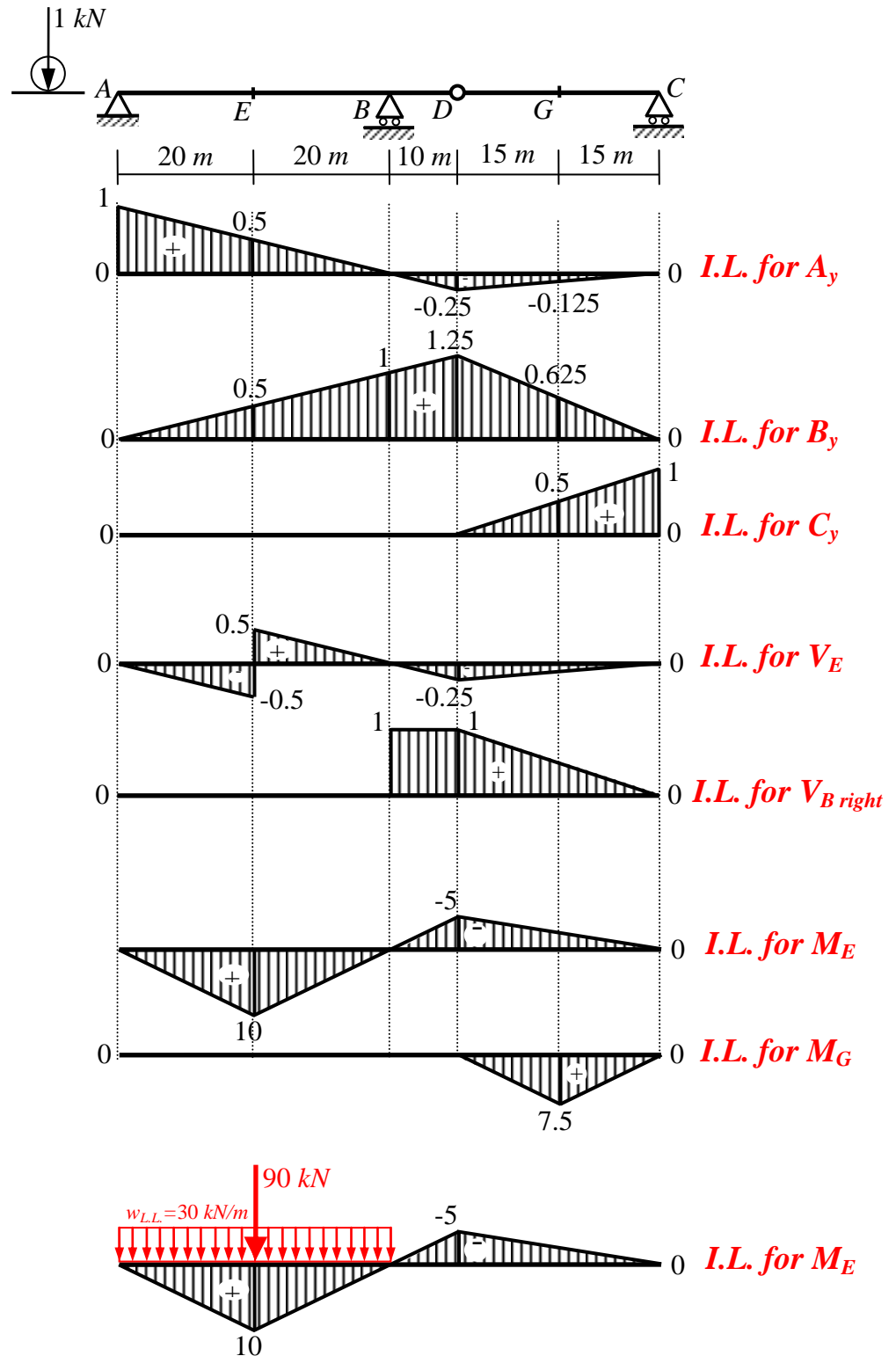
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**Question (5): (12 Marks)**

For the shown beam, draw the influence line for:

- (a) The reactions  $A_y$ ,  $B_y$  and  $C_y$ .
- (b) The shear forces at the sections  $E$  and  $B_{right}$ .
- (c) The bending moments at the sections  $E$  and  $G$ .

Also, determine the maximum moment at  $E$  caused by a concentrated moving load of  $90\text{ kN}$  and a uniform live load of  $30\text{ kN/m}$ .



For concentrated moving load,  $M_{E\max} = (90)(10) = 900\text{ kN.m} \uparrow$

For uniform live load,  $M_{E\max} = (0.5 \times 40 \times 10)(30) = 6000\text{ kN.m} \uparrow$

Total maximum moment at E,  $M_{E\max} = 900 + 6000 = \boxed{6900\text{ kN.m} \uparrow}$

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Dr. M. Abdel-Kader