Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302** Date : **9 / 5 / 2018**

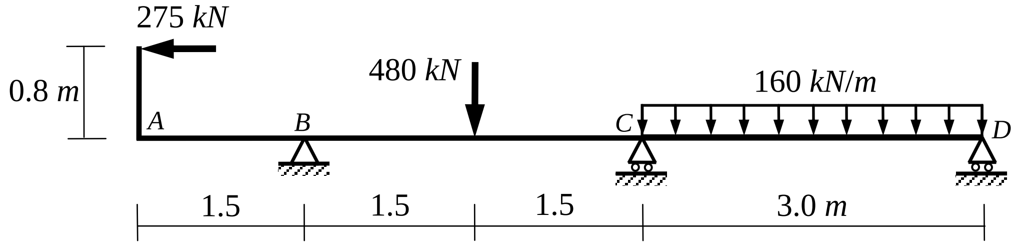
Academic Year : **2017/2018**
Semester : **Second**
Level : **3rd**
Time : **3 Hours**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Final Term Exam

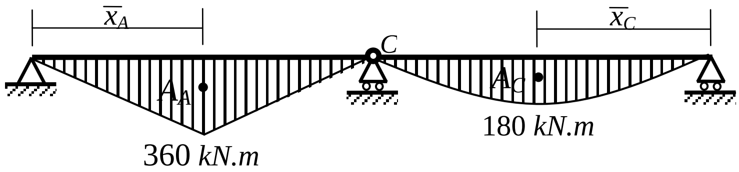
Total Marks: **60** No. of Questions: **5** (Attempt all questions)

Question (1): (12 Marks)

Using **the three-moment equation**, draw the shear force and bending moment diagrams for the shown beam.



Solution:



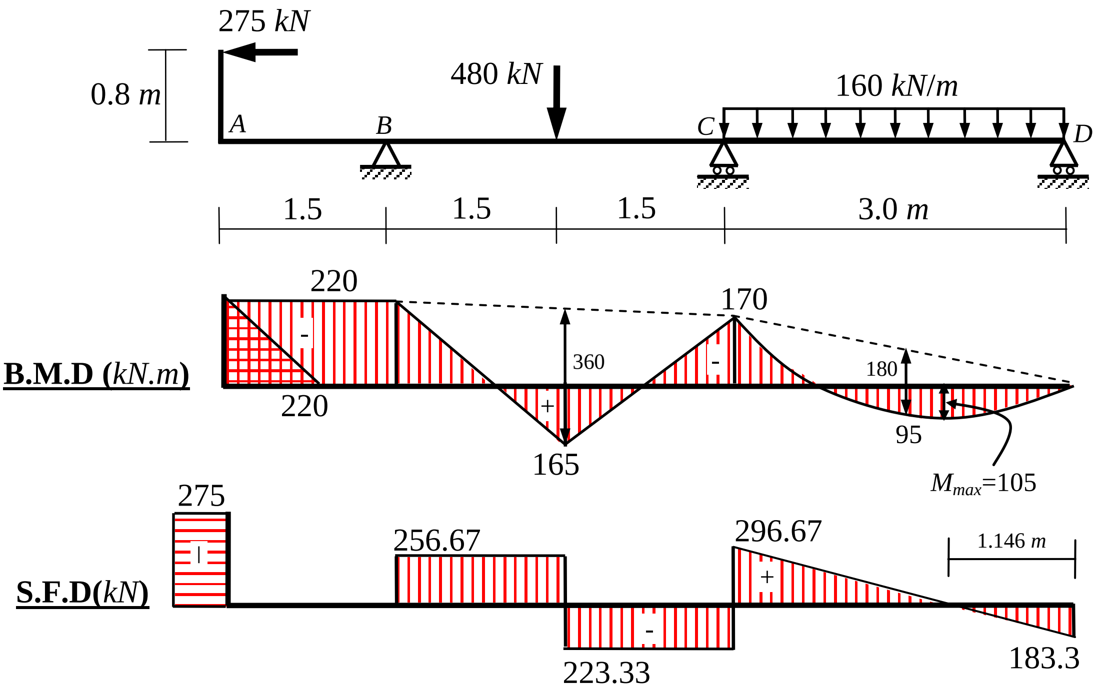
Applying three-moment equation for the spans BC and CD:

$$M_B(3) + 2M_C(3+3) + M_D(3) = -6\left(\frac{(0.5\times3\times360)1.5}{3} + \frac{(2/3\times3\times180)1.5}{3}\right)$$

$$(-220)(3) + 2M_C(6) + (0)(3) = -2700$$

$$12M_C = -2040 \rightarrow \boxed{M_C = -170\ kN.m}$$

The bending moment and shear force diagrams are shown below.



With my best wishes
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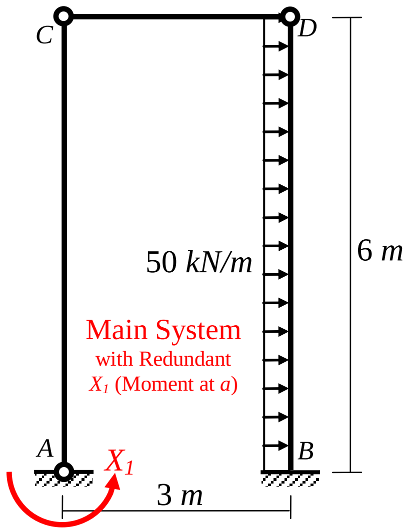
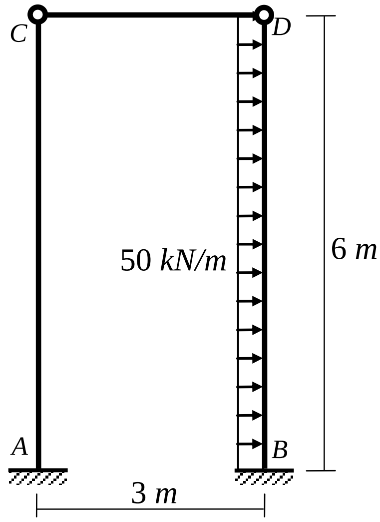
Question (2): (12 Marks)

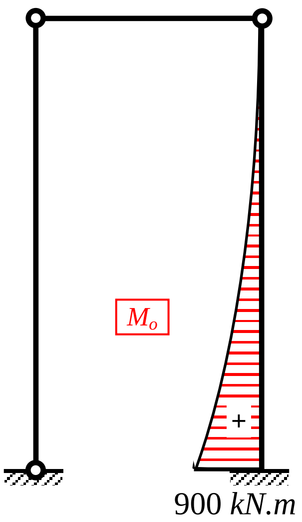
For the shown statically indeterminate frame, **using the consistent deformations (virtual work) method**, draw the bending moment diagram.

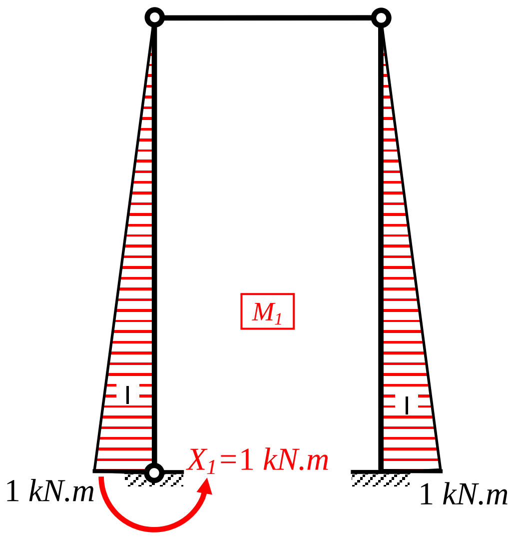
Note:

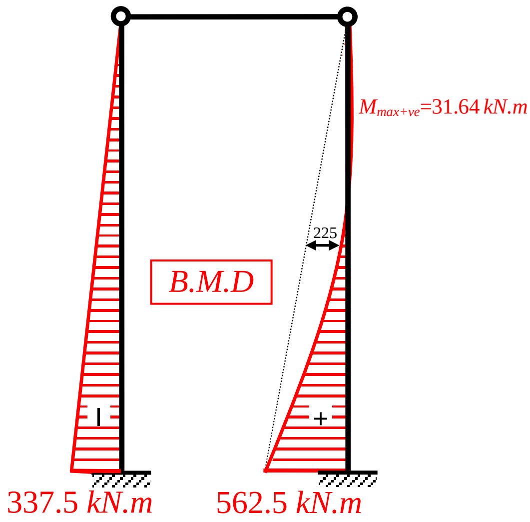
Take the main system by replacing the fixed support at *A* by a hinged support.











$$\delta_{10} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[\left(\frac{1}{3}\times 6\times 900\right)\left(-\frac{3}{4}\times 1\right)\right] = \frac{-1350}{EI}$$

$$\boxed{\delta_{10} = -1350/EI}$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL = \frac{2}{EI}\left[\left(-\frac{1}{2}\times 6\times 1\right)\left(-\frac{2}{3}\times 1\right)\right] = \frac{4}{EI}$$

$$\boxed{\delta_{11} = 4/EI}$$

$$\delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-1350}{4} = 337.5 \ kN.m$$

$$\boxed{X_1 = M_A = 337.5 \ kN.m \circlearrowleft}$$

$M_A = M_{Ao} + X_1 \ M_{a1} = 0 + (337.5)(-1) = -337.5 \ kN.m$

$M_C = M_{Co} + X_1 \ M_{c1} = 900 + (337.5)(-1) = 562.5 \ kN.m$

With my best wishes

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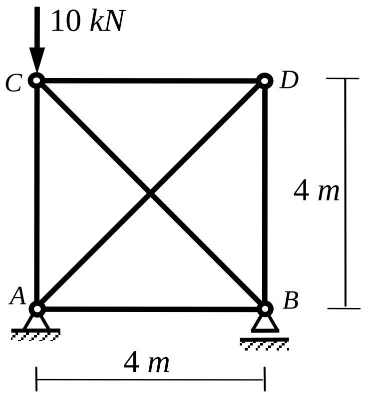
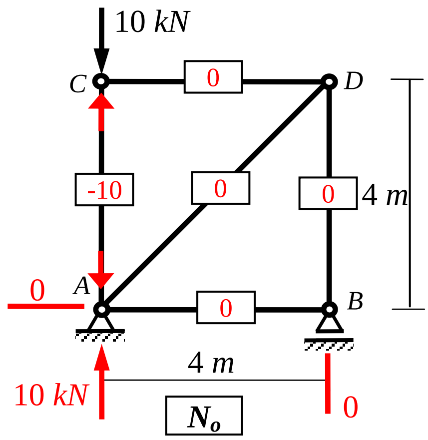
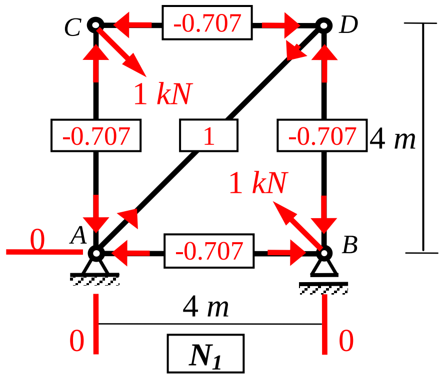
Question (3): (12 Marks)

For the shown truss, **using the consistent deformation (virtual work) method**, determine the forces in all members of the truss. Assume $EA = 1\ kN$ for all members.



Solution:





Member	N_o (kN)	N_1 (kN)	L (m)	EA (kN)	$N_o N_1 L/EA$	$N_1 N_1 L/EA$	
AB	0	-0.7071	4	1	0	2	1.04
CD	-10	-0.7071	4	1	28.2843	2	-8.96
AC	0	-0.7071	4	1	0	2	1.04
BD	0	-0.7071	4	1	0	2	1.04
AD	0	1	5.657	1	0	5.657	-1.464
BC	0	1	5.657	1	0	5.657	-1.464
Σ					$\delta_{10} = 28.2843$	$\delta_{11} = 19.314$	

$$\delta_{10} = \sum_{i=1}^{i=m} \frac{N_{oi} N_{1i} L_i}{E_i A_i} = 28.2843 \quad \text{and} \quad \delta_{11} = \sum_{i=1}^{i=m} \frac{N_{1i} N_{1i} L_i}{E_i A_i} = 19.314$$

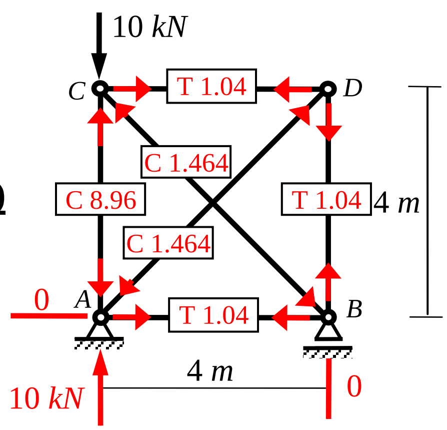
$$\delta_{CB} = \delta_{10} + X_1 \delta_{11} = 0$$

X_1 ($X_1 = F_{BC}$) can be obtained from the above equation as follows:

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} \rightarrow X_1 = -\frac{28.2843}{19.314} = -1.464\ kN$$

$$\boxed{X_1 = F_{BC} = 1.464\, kN \text{ Comp.}}$$

Forces in members (*kN*)



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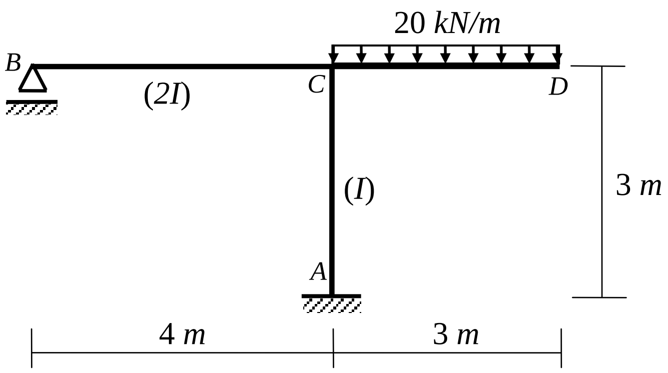
Question (4): (12 Marks)

For the shown loaded frame with variable moment of inertia, **using the slope deflection method**, draw the bending moment diagram. Note that E is constant and the relative moments of inertia are given between brackets.

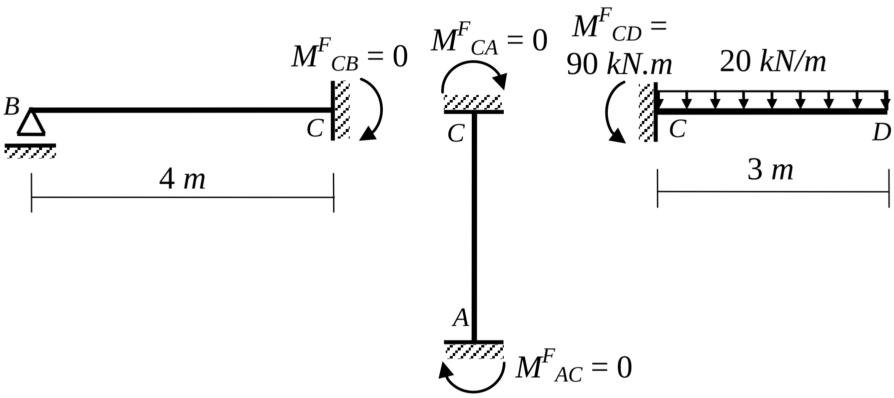


Solution:

- Unknown displacements: θ_C and Δ.
- The equilibrium equations required

$$\sum M_C = M_{CA} + M_{CB} + M_{CD} = 0$$
$$\sum F_x = 0$$

- Fixed end moments:



- The slope deflection equations are:

$$M_{CA} = M^F_{CA} + \frac{2EI}{L}\left(2\theta_C + \theta_A + 3\psi_{CA}\right) = 0 + \frac{2E(I)}{3}\left(2\theta_C + 0 - \frac{3\Delta}{3}\right) = \tfrac{4}{3}EI\theta_C - \tfrac{2}{3}EI\Delta$$

$$M_{CB} = M^F_{CB} + \frac{3EI}{L}\left(\theta_C + \psi_{CB}\right) = 0 + \frac{3E(2I)}{4}\left(\theta_C + 0\right) = \tfrac{3}{2}EI\,\theta_C$$

$$M_{CD} = -90\ kN.m$$

$$M_{AC} = M^F_{CA} + \frac{2EI}{L}\left(2\theta_A + \theta_C + 3\psi_{CA}\right) = 0 + \frac{2E(I)}{3}\left(0 + \theta_C - \frac{3\Delta}{3}\right) = \tfrac{2}{3}EI\theta_C - \tfrac{2}{3}EI\Delta$$

- Substituting into the static equilibrium equations,

$$\sum M_C = M_{CA} + M_{CB} + M_{CD} = \tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta - 90 = 0$$

$$\boxed{\tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta = 90 \quad \ldots\ldots (1)}$$

$$\sum F_x = X_A = 0$$

$$X_A = (M_{AC} + M_{CA})/L_{AC} = (\tfrac{4}{3}EI\theta_C - \tfrac{2}{3}EI\Delta + \tfrac{2}{3}EI\theta_C - \tfrac{2}{3}EI\Delta)/3 = 0$$

$$\boxed{\tfrac{2}{3}EI\theta_C - \tfrac{4}{9}EI\Delta = 0 \quad \ldots\ldots (2)}$$

- $$\tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta = 90 \quad \ldots\ldots (1)$$

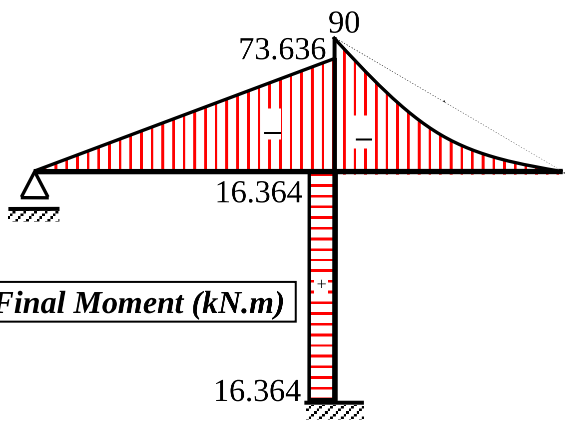
Eq.(2) $\times \frac{3}{2} \rightarrow$ $$EI\theta_C - \tfrac{2}{3}EI\Delta = 0 \quad \ldots\ldots (2^{\backslash})$$

Then $\boxed{\theta_C = \frac{540}{11EI} = \frac{49.09}{EI}}$ and $\boxed{\Delta = \frac{810}{11EI} = \frac{73.64}{EI}}$

- Back-substituting by θ_C and Δ into the slope deflection equations, the end moments become:

$$M_{CA} = \tfrac{4}{3}EI\theta_C - \tfrac{2}{3}EI\Delta = 16.364\ kN.m$$
$$M_{CB} = \tfrac{3}{2}EI\,\theta_C = 73.636\ kN.m$$
$$M_{CD} = -90\ kN.m$$
$$M_{AC} = \tfrac{2}{3}EI\theta_C - \tfrac{2}{3}EI\Delta = -16.364\ kN.m$$

- The final bending moment diagram for the whole frame is as shown.



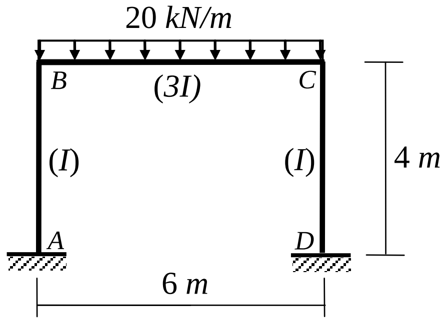
Final Moment (kN.m)

With my best wishes

Dr. M. Abdel-Kader

Question (5): (12 Marks)

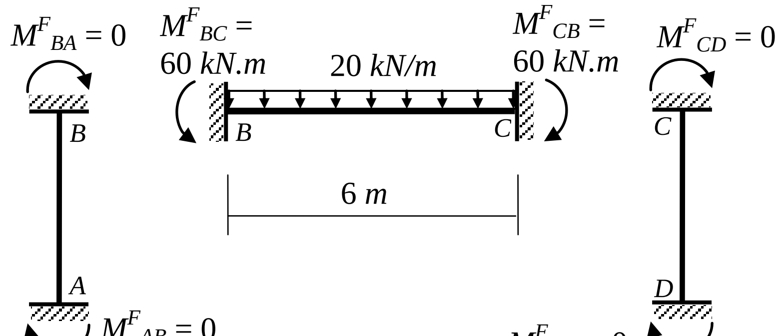
Using the **moment distribution method**, draw the bending moment diagram for the shown loaded frame with variable moment of inertia. Note that E is constant and the relative moments of inertia are given between brackets.



Solution:

1- Solution in the usual way:

- Fixed end moments:



- Distribution factors (D.F.)

At Joint *b*

$$k_{BA} = \frac{4EI}{L_{BA}} = \frac{4EI}{4} = EI$$

$$k_{BC} = \frac{4EI}{L_{BC}} = \frac{4E(3I)}{6} = 2EI$$

$$\sum k_i = k_{ba} + k_{bc} = 3EI$$

$$D.F._{BA} = \frac{k_{BA}}{\sum k_i} = \frac{1}{3} \quad \& \quad D.F._{BC} = \frac{k_{BC}}{\sum k_i} = \frac{2}{3}$$

$$D.F._{BA} = 1/3 \qquad D.F._{BC} = 2/3$$

At Joint *C*

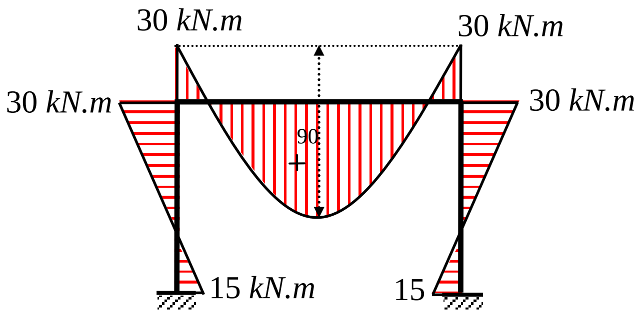
$$k_{CB} = \frac{4EI}{L_{CB}} = \frac{4E(3I)}{6} = 2EI \qquad k_{CD} = \frac{4EI}{L_{CD}} = \frac{4E(I)}{4} = EI$$

$$\sum k_i = k_{CB} + k_{CD} = 3EI$$

$$D.F._{CB} = \frac{k_{CB}}{\sum k_i} = \frac{2}{3} \quad \& \quad D.F._{CD} = \frac{k_{CD}}{\sum k_i} = \frac{1}{3}$$

$$D.F._{CB} = 2/3 \qquad D.F._{CD} = 1/3$$

Note: the sum of D.F.'s at any rigid joint =1



Bending Moment Diagram

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F.	1	1/3	2/3	2/3	1/3	1
F.E.M.	0	0	-60	+60	0	0
B.M.				-40	-20	
C.O.M.			-20			-10
B.M.		+26.67	+53.33			
C.O.M.	+13.34			+26.67		
B.M.				-17.78	-8.89	
C.O.M.			-8.89			-4.45
B.M.		+2.96	+5.93			
C.O.M.	+1.48			+2.97		
B.M.				-1.98	-0.99	
C.O.M.			-0.99			-0.5
B.M.		+0.33	+0.66			
C.O.M.	+0.17			+0.33		
B.M.				-0.22	-0.11	
M_{final}	+14.99≈ +15	29.96≈ +30	-29.96≈ -30	+29.99≈ +30	-29.99≈ -30	-14.95≈ -15

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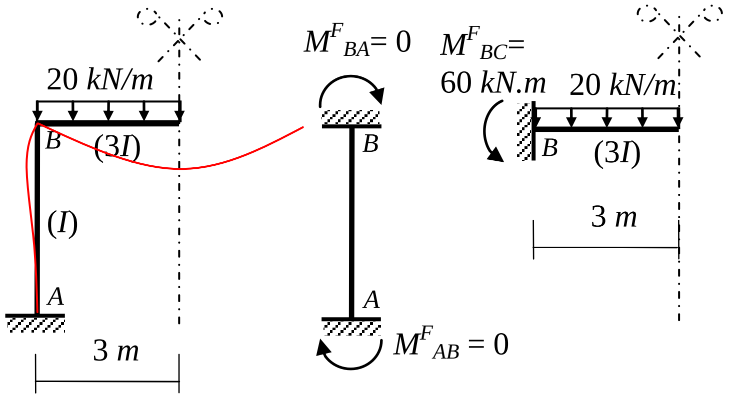
2- Solution with taken the symmetry into consideration

- Fixed end moments: (*Assume the intermediate rigid joint b to be fixed end*)



- Distribution factors (*D.F.*)

At Joint *b*

$$k_{BA} = \frac{4EI}{L_{BA}} = \frac{4EI}{4} = EI$$

$$k_{BC} = \frac{2EI}{L_{BC}} = \frac{2E(3I)}{6} = EI \qquad \sum k_i = k_{ba} + k_{bc} = 2EI$$

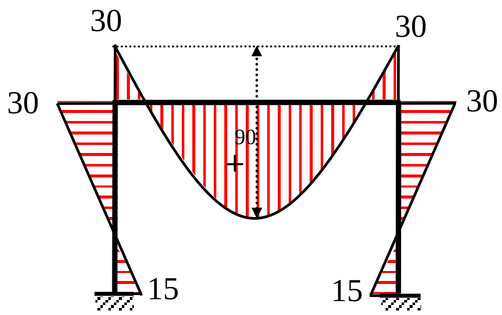
Note: the stiffness of member BC is taken 2EI/L because it is symmetrically deformed.

$$D.F._{BA} = \frac{k_{BA}}{\sum k_i} = \frac{1}{2} \quad \& \quad D.F._{BC} = \frac{k_{BC}}{\sum k_i} = \frac{1}{2}$$

$D.F._{BA} = 0.5$ $\qquad$ $D.F._{BC} = 0.5$

Note: the sum of D.F.'s at any rigid joint =1

Joint	A	B	
Member	*AB*	*BA*	*BC*
Distribution factor, D.F.	1	0.5	0.5
Fixed end moment, F.E.M.	0	0	-60
Balanced moment, B.M.		+30	+30
Carry over moment, C.O.M.	+15		
Balanced moment, B.M.			
Final bending moment, M_{final}	+15	+30	-30



B.M.D (*kN.m*)

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