

Mid-Term Exam

- The Exam consists of **2** questions in **1** page.

Question (1): (14 Marks)

For the shown frame, using the stiffness method (**neglect axial deformation**),

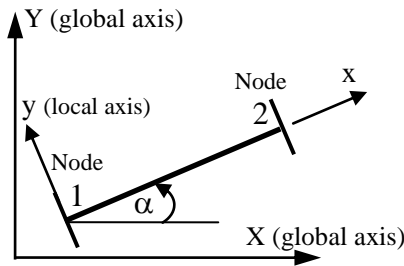
(a) determine the displacements at the nodes due to the given load.

(b) draw the bending moment diagram.

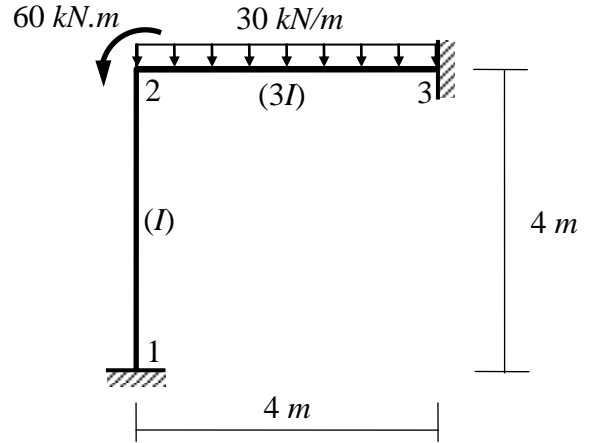
The relative moments of inertia are given between brackets.

EA is constant.

Given Data:



Frame element



$$[K_e] = \begin{bmatrix} \left(\frac{EA}{L} \lambda^2 + \frac{12EI}{L^3} \mu^2 \right) & \left(\frac{EA}{L} \mu \lambda - \frac{12EI}{L^3} \mu \lambda \right) & -\frac{6EI}{L^2} \mu & \left(-\frac{EA}{L} \lambda^2 - \frac{12EI}{L^3} \mu^2 \right) & \left(-\frac{EA}{L} \mu \lambda + \frac{12EI}{L^3} \mu \lambda \right) & -\frac{6EI}{L^2} \mu \\ \left(\frac{EA}{L} \mu \lambda - \frac{12EI}{L^3} \mu \lambda \right) & \left(\frac{EA}{L} \mu^2 + \frac{12EI}{L^3} \lambda^2 \right) & \frac{6EI}{L^2} \lambda & \left(-\frac{EA}{L} \mu \lambda + \frac{12EI}{L^3} \mu \lambda \right) & \left(-\frac{EA}{L} \mu^2 - \frac{12EI}{L^3} \lambda^2 \right) & \frac{6EI}{L^2} \lambda \\ -\frac{6EI}{L^2} \mu & \frac{6EI}{L^2} \lambda & \frac{4EI}{L} & \frac{6EI}{L^2} \mu & -\frac{6EI}{L^2} \lambda & \frac{2EI}{L} \\ \left(-\frac{EA}{L} \lambda^2 - \frac{12EI}{L^3} \mu^2 \right) & \left(-\frac{EA}{L} \mu \lambda + \frac{12EI}{L^3} \mu \lambda \right) & \frac{6EI}{L^2} \mu & \left(\frac{EA}{L} \lambda^2 + \frac{12EI}{L^3} \mu^2 \right) & \left(\frac{EA}{L} \mu \lambda - \frac{12EI}{L^3} \mu \lambda \right) & \frac{6EI}{L^2} \mu \\ \left(-\frac{EA}{L} \mu \lambda + \frac{12EI}{L^3} \mu \lambda \right) & \left(-\frac{EA}{L} \mu^2 - \frac{12EI}{L^3} \lambda^2 \right) & -\frac{6EI}{L^2} \lambda & \left(\frac{EA}{L} \mu \lambda - \frac{12EI}{L^3} \mu \lambda \right) & \left(\frac{EA}{L} \mu^2 + \frac{12EI}{L^3} \lambda^2 \right) & -\frac{6EI}{L^2} \lambda \\ -\frac{6EI}{L^2} \mu & \frac{6EI}{L^2} \lambda & \frac{2EI}{L} & \frac{6EI}{L^2} \mu & -\frac{6EI}{L^2} \lambda & \frac{4EI}{L} \end{bmatrix}$$

where, $\lambda = \cos \alpha$ and $\mu = \sin \alpha$

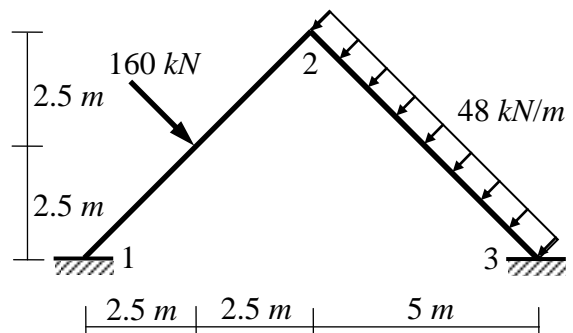
Question (2): (6 Marks)

The matrix equilibrium equation of the shown frame is:

$$\{F\} = [K] \{\Delta\} + \{F^f\}$$

Write

- the nodal forces vector $\{F\}$
- the nodal displacements vector $\{\Delta\}$
- the fixed end solution $\{F^f\}$



With my best wishes

Dr. M. Abdel-Kader