Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Computer Applications in Civil Eng.**
Course Code : **CIV 410**

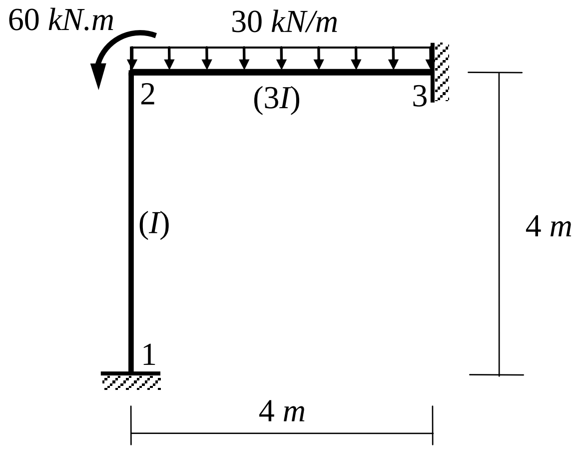
Academic Year : **2014–2015**
Semester : **Second**
Level : **4th**
Time : **1½ Hours**
Date : **1 / 4 / 2015**
Examiner: ***Dr. M. Abdel-Kader***

Mid-Term Exam

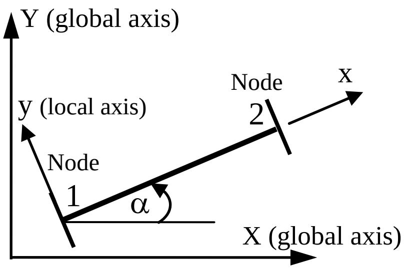
- The Exam consists of **2** questions in **1** page.

Question (1): (14 Marks)

For the shown frame, using the stiffness method (**neglect axial deformation**),
(a) determine the displacements at the nodes due to the given load.
(b) draw the bending moment diagram.
The relative moments of inertia are given between brackets.
EA is constant.



Given Data:



Frame element

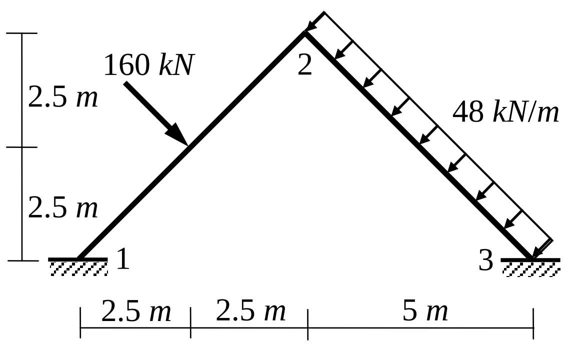
$$[K_e]=\begin{bmatrix}
\left(\frac{EA}{L}\lambda^2+\frac{12EI}{L^3}\mu^2\right) & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & -\frac{6EI}{L^2}\mu & \left(-\frac{EA}{L}\lambda^2-\frac{12EI}{L^3}\mu^2\right) & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & -\frac{6EI}{L^2}\mu \\
\left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \left(\frac{EA}{L}\mu^2+\frac{12EI}{L^3}\lambda^2\right) & \frac{6EI}{L^2}\lambda & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \left(-\frac{EA}{L}\mu^2-\frac{12EI}{L^3}\lambda^2\right) & \frac{6EI}{L^2}\lambda \\
-\frac{6EI}{L^2}\mu & \frac{6EI}{L^2}\lambda & \frac{4EI}{L} & \frac{6EI}{L^2}\mu & -\frac{6EI}{L^2}\lambda & \frac{2EI}{L} \\
\left(-\frac{EA}{L}\lambda^2-\frac{12EI}{L^3}\mu^2\right) & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \frac{6EI}{L^2}\mu & \left(\frac{EA}{L}\lambda^2+\frac{12EI}{L^3}\mu^2\right) & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \frac{6EI}{L^2}\mu \\
\left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \left(-\frac{EA}{L}\mu^2-\frac{12EI}{L^3}\lambda^2\right) & -\frac{6EI}{L^2}\lambda & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \left(\frac{EA}{L}\mu^2+\frac{12EI}{L^3}\lambda^2\right) & -\frac{6EI}{L^2}\lambda \\
-\frac{6EI}{L^2}\mu & \frac{6EI}{L^2}\lambda & \frac{2EI}{L} & \frac{6EI}{L^2}\mu & -\frac{6EI}{L^2}\lambda & \frac{4EI}{L}
\end{bmatrix}$$

where, $\lambda = \cos\alpha$ and $\mu = \sin\alpha$

Question (2): (6 Marks)

The matrix equilibrium equation of the shown frame is:

$$\{F\} = [K]\{\Delta\} + \{F^f\}$$



Write
- the nodal forces vector $\{F\}$
- the nodal displacements vector $\{\Delta\}$
- the fixed end solution $\{F^f\}$

With my best wishes
Dr. M. Abdel-Kader

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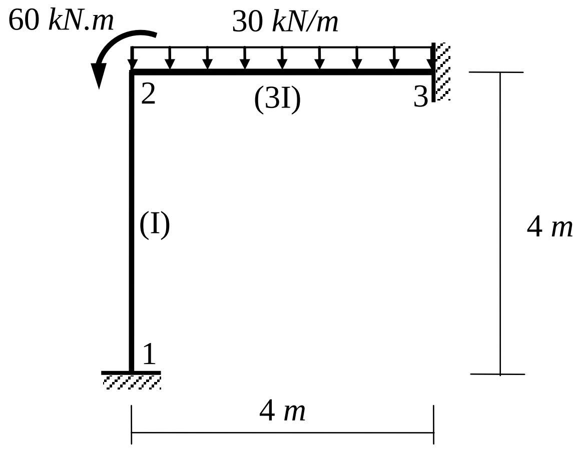


Element (1): (nodes 1 & 2)

$\alpha = 90 \quad \lambda = \cos\alpha = 0 \quad$ and $\quad \mu = \sin\alpha = 1$

$EA/L = EA/4$

$12EI/L^3 = 3EI/16$

$6EI/L^2 = 3EI/8$

$4EI/L = EI$

$2EI/L = EI/2$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix} = \begin{bmatrix} - & - & - & - & - & -3EI/8 \\ - & - & - & - & - & 0 \\ - & - & - & - & - & EI/2 \\ - & - & - & - & - & 3EI/8 \\ - & - & - & - & - & 0 \\ - & - & - & - & - & EI \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Element (2): (nodes 2 & 3)

$\alpha = 0 \quad \lambda = \cos\alpha = 1 \quad$ and $\quad \mu = \sin\alpha = 0$

$EA/L = EA/4$

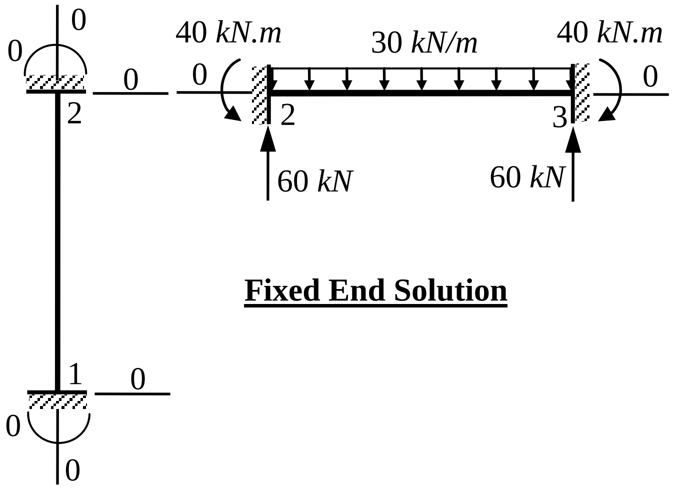
$12EI/L^3 = 9EI/16$

$6EI/L^2 = 9EI/8$

$4EI/L = 3EI$

$2EI/L = 3EI/2$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ M_2 \\ F_{x3} \\ F_{y3} \\ M_3 \end{Bmatrix} = \begin{bmatrix} - & - & 0 & - & - & - \\ - & - & 9EI/8 & - & - & - \\ - & - & 3EI & - & - & - \\ - & - & 0 & - & - & - \\ - & - & -9EI/8 & - & - & - \\ - & - & 3EI/2 & - & - & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 60 \\ 40 \\ 0 \\ 60 \\ -40 \end{Bmatrix}$$



Fixed End Solution

Frame equation

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ 0 \\ 0 \\ 60 \\ X_3 \\ Y_3 \\ M_3 \end{Bmatrix} = \begin{array}{c} \begin{array}{ccc} \quad 1 \quad\quad & \quad\quad 2 \quad\quad & \quad\quad 3 \quad \end{array} \\ \begin{bmatrix} - & - & - & - & - & -3EI/8 & - & - & - \\ - & - & - & - & - & 0 & - & - & - \\ - & - & - & - & - & EI/2 & - & - & - \\ - & - & - & - & - & (3EI/8+0) & - & - & - \\ - & - & - & - & - & (0+9EI/8) & - & - & - \\ - & - & - & - & - & (EI+3EI) & - & - & - \\ - & - & - & - & - & 0 & - & - & - \\ - & - & - & - & - & -9EI/8 & - & - & - \\ - & - & - & - & - & 3EI/2 & - & - & - \end{bmatrix} \end{array} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 60 \\ 40 \\ 0 \\ 60 \\ -40 \end{Bmatrix}$$

From Row No. 6 → $60 = (EI+3EI)(\theta_2) + 40$ → $\boxed{\theta_2 = 5/EI}$

From Element 1

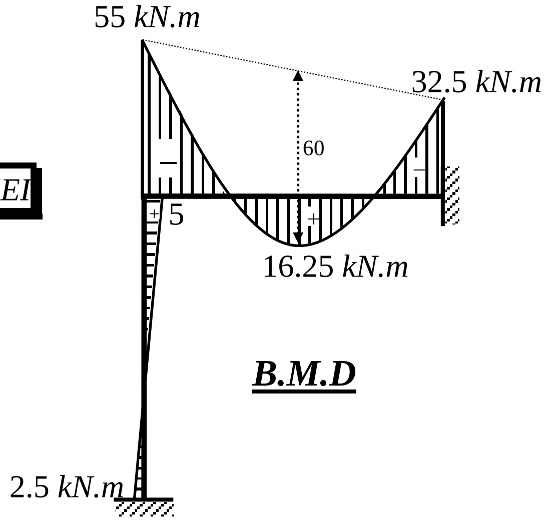
$M_1 = EI/2\,(5/EI) + 0 \quad = +2.5\ kN.m$

$M_2 = EI\,(5/EI) + 0 \quad = +5\ kN.m$

From Element 2

$M_2 = 3EI\,(5/EI) + 40 \quad = +55\ kN.m$

$M_3 = 3\,EI/2\,(5/EI) - 40 \quad = -32.5\ kN.m$



B.M.D

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The matrix equilibrium equation of the shown frame is:

$$\{F\} = [K]\,\{\Delta\} + \{F^f\}$$

Write

- the nodal forces vector $\{F\}$
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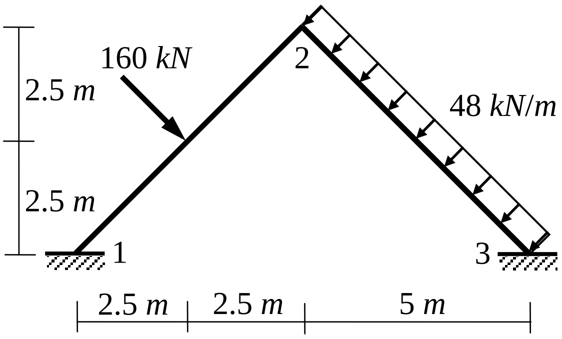
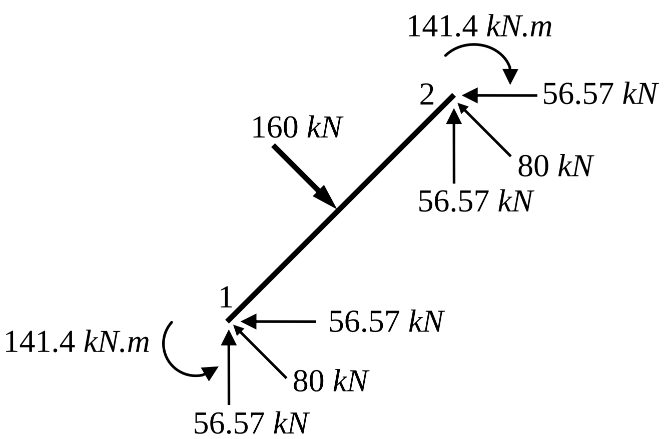
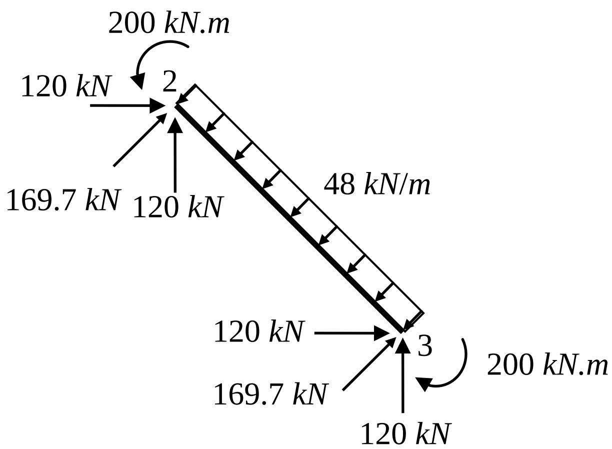
- the fixed end solution $\{F^f\}$







$$\{F\} = \begin{array}{c} \\ 1 \\ \\ \\ 2 \\ \\ \\ 3 \\ \\ \end{array} \begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ 0 \\ 0 \\ 0 \\ X_3 \\ Y_3 \\ M_3 \end{Bmatrix} \qquad \{\Delta\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ u_2 \\ v_2 \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \{F^f\} = \begin{Bmatrix} -56.57 \\ 56.57 \\ 141.4 \\ 63.43 \\ 176.57 \\ 58.6 \\ 120 \\ 120 \\ -200 \end{Bmatrix}$$

With my best wishes

Dr. M. Abdel-Kader